ECEn 682R | SOLUTIONS — HOMEWORK #1 | 9/14/2009

2.1) a) $\delta_s(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m, y-n)$

$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m)\delta(y-n)$

$= \sum_{m=-\infty}^{\infty} \delta(x-m) \sum_{n=-\infty}^{\infty} \delta(y-n)$ ✓ SEPARABLE

c) $e(x,y) = e^{j2\pi(u_0x + v_0y)} = e^{j2\pi u_0 x} e^{j2\pi v_0 y}$ ✓ SEPARABLE

d) $s(x,y) = \sin[2\pi(u_0x + v_0y)] = \sin(2\pi u_0 x + 2\pi v_0 y)$

$= \sin(2\pi u_0 x)\cos(2\pi v_0 y) + \cos(2\pi u_0 x)\sin(2\pi v_0 y)$ ...

NOT SEPARABLE

2.2) a) $\delta(x,y)$ NOT PERIODIC

b) $\mathrm{comb}(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m, y-n)$

SHIFTED BY INTEGER VALUES

PERIODIC, $X = 1$, $Y = 1$

c) $f(x,y) = \sin(2\pi x)\cos(4\pi y)$

PERIODIC, $X = 1$, $Y = \frac{1}{2}$

d) $f(x,y) = \sin(2\pi(x+y))$

PERIODIC, $X = 1$, $Y = 1$

e) $f(x,y) = \sin(2\pi(x^2+y^2))$

NOT PERIODIC

f) $f_d(m,n) = \sin\left(\frac{\pi}{5}m\right)\cos\left(\frac{\pi}{5}n\right) = \sin\left(\frac{2\pi}{10}m\right)\cos\left(\frac{2\pi}{10}n\right)$

PERIODIC, $X = 10$, $Y = 10$

g) $f_d(m,n) = \sin\left(\frac{1}{5}m\right)\cos\left(\frac{1}{5}n\right) = \sin\left(\frac{2\pi}{10\pi}m\right)\cos\left(\frac{2\pi}{10\pi}n\right)$

$\boxed{\text{NOT PERIODIC!}}$ m AND n ARE INTEGERS,

AND π IS IRRATIONAL, SO $m\pi$ OR $n\pi$

IS <u>NEVER</u> AN INTEGER MULTIPLE

OF 2π!

2.6) $g(x,y) = f(x,-1) + f(0,y)$

a) <u>LINEAR?</u>

LET: $g'(x,y)$ BE RESPONSE OF SYSTEM TO INPUT $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$.

THEN:

$$
\begin{aligned}
g'(x,y) &= f'(x,-1) + f'(0,y) \\
&= \sum_{k=1}^{K} w_k f_k(x,-1) + \sum_{k=1}^{K} w_k f_k(0,y) \\
&= \sum_{k=1}^{K} \left[w_k f_k(x,-1) + w_k f_k(0,y) \right] \\
&= \sum_{k=1}^{K} w_k \left[f_k(x,-1) + f_k(0,y) \right] \\
&= \sum_{k=1}^{K} w_k g_k(x,y) \quad \checkmark
\end{aligned}
$$

$\boxed{\text{LINEAR}}$

b) <u>SHIFT INVARIANT?</u>

LET: $g'(x,y)$ BE RESPONSE OF SYSTEM TO $f'(x,y) = f(x-x_0, y-y_0)$.

THEN:

$$g'(x,y) = f'(x,-1) + f'(0,y)$$

$$g'(x,y) = f(x-x_0, -1-y_0) + f(0-x_0, y-y_0) \neq g(x-x_0, y-y_0)$$

$$g(x-x_0, y-y_0) = f(x-x_0, -1) + f(0, y-y_0)$$

$\boxed{\text{NOT SHIFT INVARIANT!}}$

2.7) a) $g(x,y) = f(x,y)\,f(x-x_0,y)$

LINEAR? LET: $f'(x,y) = \sum_k \omega_k f_k(x,y)$

$$g'(x,y) = f'(x,y)\,f'(x-x_0,y)$$

$$= \sum_{k=1}^{\infty} \omega_k f_k(x,y) \sum_{\ell=1}^{\infty} w_\ell f_\ell(x-x_0,y)$$

IS THIS THE SAME AS $\sum_{k=1}^{\infty} \omega_k g_k(x,y)$??

NO! WE'VE GOT ALL KINDS OF CROSS TERMS!

$\boxed{\text{NOT LINEAR}}$

SHIFT INVARIANT?

LET: $f'(x,y) = f(x-x_0,y-y_0)$

$$g'(x,y) = f'(x,y)\,f'(x-x_0,y)$$

$$= f(x-x_0,y-y_0)\,f(x-x_0-x_0,y-y_0)$$

$$= f(x-x_0,y-y_0)\,f(x-2x_0,y-y_0) = g(x-x_0,y-y_0)$$

YES, SO $\boxed{\text{SHIFT INVARIANT}}$

$$g(x-x_0,y-y_0) = f(x-x_0,y-y_0)\,f(x-x_0-x_0,y-y_0)$$

$$= f(x-x_0,y-y_0)\,f(x-2x_0,y-y_0)$$

b) $g(x,y) = \int_{-\infty}^{\infty} f(x,\eta)\,d\eta$

LINEAR? LET $f'(x,y) = \sum_k \omega_k f_k(x,y)$

THEN:

$$g'(x,y) = \int_{-\infty}^{\infty} f'(x,\eta)\,d\eta = \int_{-\infty}^{\infty} \left[\sum_{k=1}^{\infty} w_k f_k(x,\eta)\right] d\eta$$

$$= \sum_{k=1}^{\infty} \omega_k \left[\int_{-\infty}^{\infty} f_k(x,\eta)\,d\eta\right] = \sum_{k=1}^{\infty} \omega_k g_k(x,y) \checkmark$$

$\boxed{\text{LINEAR}}$

SHIFT INVARIANT? LET: $f'(x,y) = f(x-x_0,y-y_0)$

THEN:

$$g'(x,y) = \int_{-\infty}^{\infty} f'(x,\eta)\,d\eta = \int_{-\infty}^{\infty} f(x-x_0,\eta-y_0)\,d\eta = g(x-x_0,y-y_0)$$

$$g(x-x_0,y-y_0) = \int_{-\infty}^{\infty} f(x-x_0,\eta)\,d\eta = \int_{-\infty}^{\infty} f(x-x_0,\eta'-y_0)\,d\eta'$$

$\boxed{\text{SHIFT INVARIANT!}}$

2.8)

a) $h(x,y) = x^2 + y^2$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |h(x,y)|\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |x^2+y^2|\,dx\,dy = \infty$$

NOT STABLE

b) $h(x,y) = e^{-(x^2+y^2)}$

$$\iint_{-\infty}^{\infty} |e^{-(x^2+y^2)}|\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{\infty} e^{-x^2}dx \int_{-\infty}^{\infty} e^{-y^2}dy = \sqrt{\pi}\cdot\sqrt{\pi} = \boxed{\pi} < \infty$$

STABLE

c) $h(x,y) = x^2 e^{-y^2}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |h(x,y)|\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 e^{-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{\infty} x^2\,dx \underbrace{\int_{-\infty}^{\infty} e^{-y^2}dy} = \sqrt{\pi}\int_{-\infty}^{\infty} x^2 dx = \infty$$

NOT STABLE

2.9) $g(x) = f(x) * f(x)$

a) $$\boxed{g(x) = \int_{-\infty}^{\infty} f(\xi) f(x-\xi)\,d\xi}$$

b) LINEAR? LET: $f' = \sum_{k=1}^{N} \omega_k f_k(x)$

$$g'(x) = f'(x) * f'(x) = \int_{-\infty}^{\infty} f'(\xi) f'(x-\xi)\,d\xi = \int_{-\infty}^{\infty}\left(\sum_{k=1}^{\infty} \omega_k f_k(\xi)\right)\left(\sum_{l=1}^{\infty} \omega_l f_l(x-\xi)\right)d\xi$$

$$= \sum_{k}\sum_{l}\int_{-\infty}^{\infty} \omega_k \omega_l f_k(\xi) f_l(x-\xi)\,d\xi$$

$$\neq \sum_{k} \omega_k g_k(x)$$

NOT LINEAR

c) SHIFT INVARIANT?

LET: $f'(x) = f(x-x_0)$

THEN:

$$g'(x) = f'(x) * f'(x) = \int_{-\infty}^{\infty} f'(\xi) f'(x-\xi)\, d\xi$$

$$= \int_{-\infty}^{\infty} f(\xi - x_0) f(x-\xi-x_0)\, d\xi$$

$$= \int_{-\infty}^{\infty} f(\xi') f(x-\xi')\, d\xi' \neq g(x-x_0)$$

?

NOT SHIFT INVARIANT!

$$g(x-x_0) = f(x-x_0) * f(x-x_0) = \int_{-\infty}^{\infty} f(\xi) f(x-x_0-\xi)\, d\xi$$

2.10) $f(x,y) = x + y^2$

(a) $f(x,y)\delta(x-1,y-2) = f(1,2)\delta(x-1,y-2)$

$$= (1+2^2)\delta(x-1,y-2) = \boxed{5\delta(x-1,y-2)}$$

(b) $f(x,y) * \delta(x-1,y-2) = f(x-1,y-2) = x-1+(y-2)^2$

$$= x-1+y^2-4y+4 = \boxed{x+y^2-4y+3}$$

(c) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x-1,y-2) f(x,3)\, dx\, dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x-1,y-2)(x+9)\, dx\, dy$$

$$= 1+9 = \boxed{10}$$ SIFTING PROPERTY

OR, EQUIVALENTLY:

$$\iint_{-\infty}^{\infty} \delta(x-1,y-2) f(x,3)\, dx\, dy = f(1,3) = 1+3^2 = 1+9 = 10$$

(d) $\delta(x-1,y-2) * f(x+1,y+2) = f(x,y) = \boxed{x+y^2}$

SHIFTED −1 IN x, −2 IN y

SHIFTS IT BACK TO ORIGIN!