ECEN 682R	SOLUTIONS HOMEWORK #2	9/28/2009

2.14) a) $f(x)$ IS <u>REAL</u>, SO $f^*(x) = f(x)$.

THEN WE HAVE:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$ BY DEFINITION OF F.T.

AND:

$$F^*(u) = \left[\int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx\right]^* = \int_{-\infty}^{\infty} \underbrace{f^*(x)}_{=f(x)} e^{j2\pi ux} dx$$

$$= \int_{-\infty}^{\infty} f(x) e^{j2\pi ux} dx$$

CHANGE OF VARIABLES

LET: $x' = -x \quad x = -x'$

THEN: $dx' = -dx \quad dx = -dx'$

$$= \int_{\infty}^{-\infty} f(-x') e^{-j2\pi ux'} (-dx')$$

$$= -\int_{\infty}^{-\infty} f(-x') e^{-j2\pi ux'} dx'$$

$$= \int_{-\infty}^{\infty} f(-x') e^{-j2\pi ux'} dx'$$

$f(-x) = f(x)$, SO:

$$= \int_{-\infty}^{\infty} f(x') e^{-j2\pi ux'} dx' = F(u) \checkmark$$

b) <u>SAME AS a), UP UNTIL ~~HERE~~:</u>

$f(-x) = -f(x)$, SO:

$$F^*(u) = \int_{-\infty}^{\infty} -f(x') e^{-j2\pi ux'} dx' = -F(u) \checkmark$$

2.18) $f(x,y) = \sin(2\pi a x)\cos(2\pi b y)$

USE THE FACT THAT $F(u,v) = F_1(u)F_2(v)$
IF $f(x,y) = f_1(x)f_2(y)$

WE HAVE:

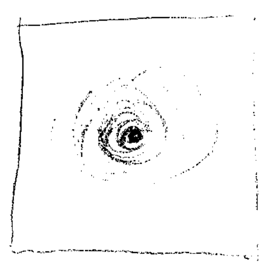
$$F(u,v) = \frac{1}{2j}\left[\delta(u-a) - \delta(u+a)\right]\frac{1}{2}\left[\delta(v-b) + \delta(v+b)\right]$$

$$= \frac{1}{4j}\left[\delta(u-a)\delta(v-b) + \delta(u-a)\delta(v+b) - \delta(u+a)\delta(v-b) - \delta(u+a)\delta(v+b)\right]$$

$$\boxed{F(u,v) = \frac{1}{4j}\left[\delta(u-a,v-b) + \delta(u-a,v+b) - \delta(u+a,v-b) - \delta(u+a,v+b)\right]}$$

2.21) $h(x,y) = e^{-\pi(x^2+y^2)/4}$ $\Leftarrow$ NOTE: IS THE SYSTEM REALLY ANISOTROPIC AS THEY CLAIM? (I THINK THEY MEANT: $h(x,y) = e^{-\pi(x^2 + y^2/4)}$) :)

a) 2D GAUSSIAN



$\Leftarrow$ PLOT IN MATLAB TO SEE A BETTER VERSION. :)

b) $H(u,v) = \mathcal{F}_{2D}\left\{e^{-\pi(x^2+y^2)/4}\right\} = \mathcal{F}_{2D}\left\{e^{-\pi\left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2\right)}\right\}$

$$= \frac{1}{\left|\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right|} e^{-\pi\left((2u)^2 + (2v)^2\right)} = \boxed{4e^{-4\pi(u^2+v^2)}}$$

SCALING PROPERTY OF FOURIER TRANSFORM w/ $a = \frac{1}{2}$, $b = \frac{1}{2}$

2.25) $f(x,y) = e^{-\pi(x^2+y^2)}$

SAMPLE AT 1.5 samples/mm.

SO:

$$\frac{1}{\Delta y} = \frac{1}{\Delta x} = 1.5 \text{ mm}^{-1}$$

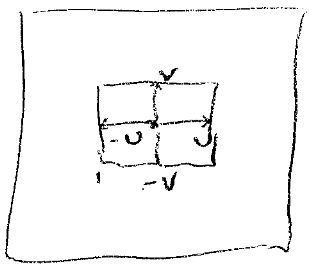
$$\Delta y = \Delta x = \frac{1}{1.5}\text{mm} = \frac{2}{3}\text{ mm}$$

SO OUR LOW-PASS FILTER NEEDS:

$$U \le \frac{1}{2\Delta x} \qquad U \le \frac{1}{2\cdot\frac{2}{3}} = \frac{3}{4}\text{ mm}^{-1}$$

$$V \le \frac{1}{2\Delta y} \qquad V \le \frac{1}{2\cdot\frac{2}{3}} = \frac{3}{4}\text{ mm}^{-1}$$

SO: $H(u,v) =$



$$H(u,v) = \begin{cases} 1, & |u| \le 3/4 \text{ AND } |v| \le 3/4 \\ 0, & \text{else} \end{cases}$$

OR:

$$H(u,v) = \text{rect}\left(\frac{2u}{3}, \frac{2v}{3}\right)$$

WE WANT $h(x,y)$ (THE PSF), SO WE NEED TO TAKE INVERSE 2DFT:

$$h(x,y) = \mathcal{F}_{2D}^{-1}\{H(u,v)\} = \mathcal{F}_{2D}^{-1}\left\{\text{rect}\left(\frac{2u}{3}, \frac{2v}{3}\right)\right\}$$

$$h(x,y) = \left|\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\right| \text{sinc}\left(\frac{3}{2}x, \frac{3}{2}y\right)$$

$$\boxed{h(x,y) = \frac{9}{4}\text{sinc}\left(\frac{3}{2}x, \frac{3}{2}y\right)}$$

EITHER IS OKAY.

OR:

$$h(x,y) = \frac{\cancel{\frac{9}{4}}\sin\left(\frac{3}{2}\pi x, \frac{3}{2}\pi y\right)}{\pi\frac{3}{2}x\cdot\pi\frac{3}{2}y} = \boxed{\frac{\sin\left(\frac{3\pi x}{2}, \frac{3\pi y}{2}\right)}{\pi^2 x y}}$$

NEXT PART: WHAT PERCENTAGE OF SPECTRUM ENERGY IS PRESERVED BY THIS FILTER?

$$\text{PRESERVED SPECTRAL ENERGY} = \frac{\int_{-.75}^{.75}\int_{-.75}^{.75} \left|e^{-\pi(x^2+y^2)}\right|^2 dx\,dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|e^{-\pi(x^2+y^2)}\right|^2 dx\,dy}$$

$$= \frac{\int_{-.75}^{.75}\int_{-.75}^{.75} e^{-2\pi(x^2+y^2)} dx\,dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-2\pi(x^2+y^2)} dx\,dy}$$

$$= \frac{.4922}{.5} = \boxed{98.4\% \text{ SPECTRAL ENERGY PRESERVED!}}$$

HOW TO DO THE INTEGRALS IN MATLAB:

$$\int_{-.75}^{.75}\int_{-.75}^{.75} e^{-2\pi(x^2+y^2)} dx\,dy = \left(\int_{-.75}^{.75} e^{-2\pi x^2} dx\right)\left(\int_{-.75}^{.75} e^{-2\pi y^2} dy\right) = \left[\int_{-.75}^{.75} e^{-2\pi x^2} dx\right]^2$$

IN MATLAB:

```
syms x;
eval(int(exp(-2*pi*x^2),-.75,.75))^2
```

$\boxed{\text{THIS GIVES .4922}}$

```
eval(int(exp(-2*pi*x^2),-inf,inf))^2
```

$\boxed{\text{THIS GIVES 0.5}}$

CAN WE SAMPLE $f(x,y)$ ALIAS-FREE WITHOUT A FILTER?

$\boxed{\text{NO! } f(x,y) = e^{-\pi(x^2+y^2)} \text{ IS NOT BAND LIMITED.}}$

2.28) a) $H(u) = 1 - \text{rect}\left(\frac{u}{2U_0}\right)$

THIS IS A RECT THAT GOES FROM $u=-U_0$ TO U_0, SINCE $\text{rect}(u)$ GOES FROM $u=-\frac{1}{2}$ TO $\frac{1}{2}$.

$$h(t) = \mathcal{F}^{-1}\{H(u)\} = \mathcal{F}^{-1}\left\{1 - \text{rect}\left(\frac{u}{2U_0}\right)\right\}$$

$$\boxed{h(t) = \delta(t) - 2U_0 \,\text{sinc}(2U_0 t)}$$

OR

$$h(t) = \delta(t) - \cancel{2U_0}\,\frac{\sin(2\pi U_0 t)}{\cancel{2\pi U_0}\, t}$$

EITHER ONE OKAY

$$\boxed{h(t) = \delta(t) - \frac{\sin(2\pi U_0 t)}{\pi t}}$$

b) PART 1:

$f(t) = c$ $\qquad g(t) = f(t) * h(t)$

$$F(u) = c\,\delta(u) \longrightarrow G(u) = F(u)H(u)$$

$$= c\,\delta(u)H(u) = 0!$$

THIS IS ZERO EVERYWHERE BUT $u=0$

THIS IS ZERO AT $u=0$ FROM GRAPH

SO: $G(u) = 0$

AND $\boxed{g(t) = 0}$

MATLAB EXERCISE: HIGH-PASS IMAGE