Homework #6

Special Topics in Signals & Systems: Biomedical Imaging
ECEn 682R, Section 3

Due: Thursday 11/5/2009 by midnight in the box outside Dr. Bangerter's office.

Homework help sessions: I will be holding a homework help session (in addition to my regularly scheduled office hours) for each homework assignment. For Homework #6, the help session will be Tuesday 11/3 from 5-6pm in 490 CB. If you need help and cannot make the help session, please see me during office hours or contact me to arrange an alternate time.

Problem 1:

A material has equilibrium magnetization M_0 (in the *z* direction) and relaxation time constants T_1 and T_2. If a 90 degree excitation is applied:

(a) Find an expression for $|\mathbf{M}(t)|$, the magnitude of the magnetization as a function of time.
(b) Show that if $T_2 < T_1$, $|\mathbf{M}(t)|$ can never exceed M_0.

Problem 2:

Consider two tissues A and B with the same equilibrium magnetization M_0 (again in the *z* direction) but with relaxation time constants T_{1A} and T_{2A} for tissue A and T_{1B} and T_{2B} for tissue B. Assume that our pulse sequence excites both tissues with a 90 degree excitation, and that $t = 0$ immediately after the excitation.

(a) We want to determine how long after excitation we should wait before sampling our signal in order to maximize the CNR (contrast-to-noise ratio) between tissues A and B. Find an expression for the time t_d that maximizes CNR between the two tissues. (Recall that CNR is defined as the absolute difference in signal between the two tissues divided by the standard deviation of the noise. Assume that the standard deviation of the noise σ_N remains constant in time.)

Your expression in part (a) should be dependent only on the T_2 values of each tissue. Instead of generating contrast based on T_2, we now decide to generate contrast based on T_1. We do this by applying a *second* 90 degree excitation immediately prior to sampling the signal.

(b) Find an expression for the time t_{prep} between the first and second 90 degree excitations that maximizes CNR between the two tissues. Assume that we sample the signal *immediately* after the second 90 degree excitation.

(c) Now assume that tissue A is white matter (T_{1A} = 780 ms, T_{2A} = 92 ms) and tissue B is gray matter (T_{1B} = 920 ms, T_{2B} = 100 ms). Evaluate your expressions for t_d and t_{prep}.

(d) Using Matlab, plot the gray matter/white matter CNR as a function of t_d for t_d values ranging from 0 to 500 ms. Assume $\sigma_N = 1$. Are your results consistent with part (c)? Explain.

(e) Now plot the gray matter/white matter CNR as a function of t_{prep} for t_{prep} values ranging from 0 to 5000 ms. Assume $\sigma_N = 1$. Are your results consistent with part (c)? Explain.

(f) Which of the two schemes is better for maximizing gray matter/white matter CNR?

You are curious whether you can further increase CNR by doing a hybrid between the two techniques above. You apply a 90 degree excitation, wait a time t_{prep}, apply a second 90 degree excitation, and then wait a time t_d prior to sampling. (Note that this is the same as our second scheme above when $t_d = 0$.)

(g) Write an expression for the CNR as a function of both t_{prep} and t_d given the new scheme.

(h) Can you get better gray matter/white matter CNR using this scheme?

Problem 3:

Spatial resolution in MRI can be defined as half of the period of the highest spatial frequency sampled in each direction. In other words, if k_{xmax} is the highest spatial frequency acquired in the k_x direction, the x-resolution δ_x is given by:

$$\delta_x = \frac{1}{2k_{xmax}}$$

Your MRI scanner has an x-gradient G_x capable of a maximum gradient amplitude of 4 Gauss/cm and a y-gradient G_y capable of a maximum gradient amplitude of 3 Gauss/cm.

(a) If you require your signal readout (i.e., the period over which you sample the signal) to end no more than 8 ms after excitation (to avoid significant T_2 decay), what is the highest resolution you can achieve (in the x-direction and the y-direction) when imaging hydrogen (^{1}H, $\gamma/2\pi$ = 42.575 MHz/Tesla)? (Assume that we can turn on our gradient and begin readout instantaneously after excitation.)

If your samples in k-space in the k_x direction are separated by Δ_{k_x} then your field of view (FOV) in the x-direction is given by:

$$\text{FOV}_x = \frac{1}{\Delta_{k_x}}$$

(b) If you sample the signal every 4 μs while maximizing your resolution, what will your FOV be in both the x-direction and the y-direction?

(c) Repeat part (a) when imaging sodium (^{23}Na, $\gamma/2\pi$ = 11.262 MHz/Tesla).

Problem 4:

Consider a pulse sequence that performs a 90 degree excitation every 40 ms. The MR signal is sampled very rapidly immediately after each excitation (before any significant relaxation can occur). Using Matlab, compute and plot the MR signal for both gray matter and white matter for the first 200 excitations. Approximately how long does it take before our signal is in steady state?

Assume equilibrium magnetization $M_0 = 1$ (in the z direction) for both gray matter and white matter, and use the T_1 and T_2 values given in Problem 2.

Hint: Include both T1 and T2 relaxation, and realize that we are re-tipping our magnetization long before it has recovered to thermal equilibrium. Each time we do a 90 degree excitation, our longitudinal magnetization M_z is tipped down into the xy-plane, and our M_{xy} is tipped along the negative z-axis.