Homework #7

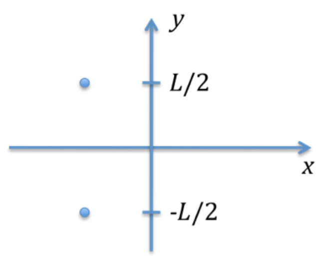
Special Topics in Signals & Systems: Biomedical Imaging
ECEn 682R, Section 3

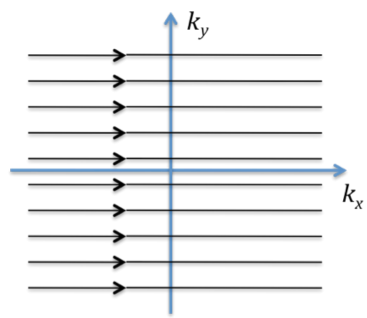
Due: Tuesday 11/17/2009 by midnight in the box outside Dr. Bangerter's office.

Homework help sessions: I will be holding a homework help session (in addition to my regularly scheduled office hours) for each homework assignment. For Homework #7, the help session will be Thursday 11/12 from 5-6pm in 490 CB. If you need help and cannot make the help session, please see me during office hours or contact me to arrange an alternate time.

Problem 1:

An object consists of two impulses (delta functions) separated in the *y*-direction by distance *L*, and is imaged using a 2DFT sequence with a *k*-space trajectory as shown below:





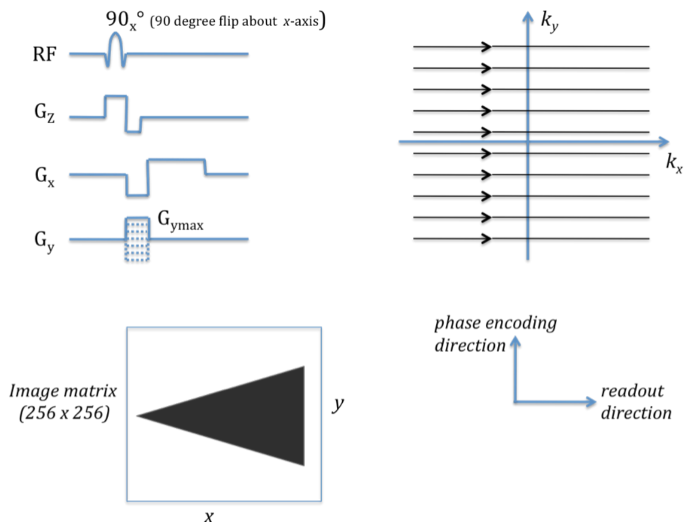
(a) At what maximum FOV in the *y*-direction will the image equal zero?

(b) At what FOV in the *y*-direction will the image be a single impulse?

(c) For parts (a) and (b), does it matter that the two impulses are centered about the *x* axis?

Problem 2:

A 2DFT imaging sequence is executed in which *k*-space is acquired in a raster-line fashion, beginning with the most negative k_y phase encode and moving progressively to the adjacent phase encode. 256 phase encodes symmetrically spaced about the origin are collected, with each readout time signal sampled to 256 points after passing through a low-pass (anti-aliasing) filter appropriate for the sampling rate. The inflexible computer is programmed to simply take the incoming data, fill a 256 by 256 matrix from the bottom row to the top row, and then perform a 2D Inverse FFT of this matrix to reconstruct the 256 by 256 magnitude image matrix as shown at the top of the following page. Assume that we wait long enough between each repetition (that is, between each excitation/phase-encoding/readout step) that all magnetization has returned to thermal equilibrium.



For each of the following modifications (a) - (g) to the 2DFT sequence described above, sketch the resultant 256 by 256 magnitude image matrix. You may clarify your answer with words if you wish.

(a) The maximum phase-encode amplitude is cut in half, decreasing from G_{ymax} to $G_{ymax}/2$ (but 256 phase encodes are still performed).

(b) The G_x gradient is inverted (that is, G_x is replaced by $-G_x$).

(c) The low-pass (anti-aliasing) filter bandwidth is cut in half while keeping the time sampling rate the same.

(d) Instead of applying a $90_x°$ tip each excitation, we apply $90_x°$, $-90_x°$, $90_x°$, $-90_x°$, … as we progress through the phase encodes. (That is, the first phase encode we tip by 90 degrees about the *x*-axis, the second phase encode we tip by -90 degrees about *x*, and continue alternating between 90 and -90 degrees on subsequent phase encodes.)

(e) Instead of applying a $90_x°$ tip each excitation, we apply $90_x°$, $90_y°$, $-90_x°$, $-90_y°$, … as we progress through the phase encodes.

(f) The G_x gradient amplitude is multiplied by 2.

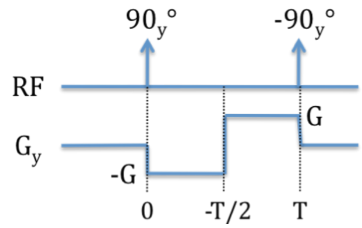
(g) The image is reconstructed by using a 2D FFT instead of an inverse 2D FFT.

Problem 3:

Consider the sequence shown below in which a $90_y°$ pulse, a zero-area gradient waveform, and finally a second $-90_y°$ pulse is applied. Immediately after the initial $90_y°$ excitation, an impulse object (with equilibrium magnetization M_0), positioned at $(0, y_0)$, moves with constant velocity v in the *y*-direction. Note that in this situation, the relative frequency of the moving impulse object is given by:

$$f(t) = \frac{\gamma}{2\pi} G_y(t)(y_0 + vt)$$

Thus, the frequency of the moving impulse object depends on both the time-varying gradient (as it would if the impulse were static) *and* the motion, since the object is moving to different *y* locations while the G_y gradient is turned on. ***Ignore T_1 and T_2 effects in this problem.***



(a) Determine the resultant transverse magnetization M_{xy} (both magnitude and phase) of the moving spin just *prior* to the second excitation pulse.

(b) Determine the resultant transverse magnetization M_{xy} (both magnitude and phase) of the moving spin immediately *after* the second excitation pulse.

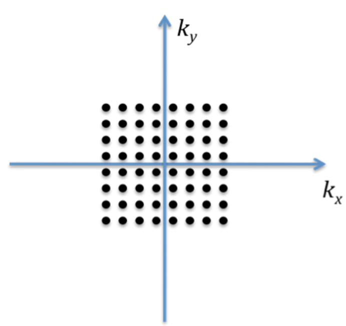
(c) Determine the smallest velocity ($v > 0$) that will result in zero signal.

(d) If G = 0.3 G/cm, T = 4 ms, and v = 20 cm/s, should the second excitation pulse be $-90_y°$ (i.e., should it be a negative 90 degree pulse around the y-axis) in order to produce the maximum MR signal? If not, around what direction should the second -90° pulse be performed (e.g., the x-axis, the negative x-axis, the negative y-axis, or some direction in between)? (You can sketch this in the rotating frame to clarify which direction we should tip the resulting magnetization in order to maximize signal.)

Problem 4:

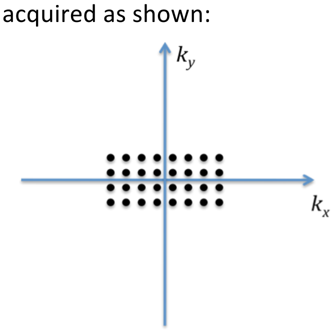
A reference 2DFT sequence is as follows:

- 256 phase encodes are performed
- The readout duration is 10 ms per measurement (i.e., per phase encode step), sampled to 256 points
- Field of view is FOV_x in the x-direction and FOV_y in the y-direction
- Spatial resolution is δ_x in the x-direction and δ_y in the y-direction
- k-space coverage is as shown below:

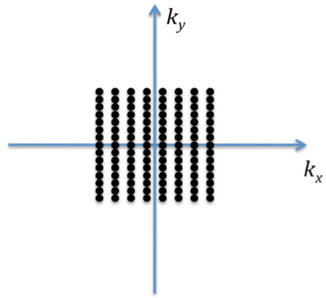


If this sequence produces an SNR normalized to 1, then what are the relative SNR's of each of the following sequences? Assume that the readout gradient amplitude does not change compared to the reference sequence. Comment on the relative FOV's, spatial resolution, and signal bandwidth in each case.

(a) Only half the phase-encodes are acquired as shown:



(b) Twice as many phase encodes are acquired (effectively decreasing the distance between phase encodes in the k_y direction by a factor of 2) as shown:



(c) We push out twice as far in all directions in k-space as shown below, but keep the number of phase encodes at 256. (Remember that in order to achieve twice the k-space coverage in the k_x direction without changing the readout gradient amplitude, we will have to double our readout duration.)