PROBLEM 1:

THERE ARE TWO ANSWERS I'LL ACCEPT FOR THIS PROBLEM.

THE FIRST IS TECHNICALLY INCORRECT GIVEN THE DIAGRAM I INCLUDED IN THE HOMEWORK SET (THANK YOU TO A VERY TALENTED STUDENT FOR POINTING THAT OUT), BUT IT IS THE SOLUTION WE DISCUSSED IN THE HOMEWORK HELP SESSION. MY APOLOGIES.

OUR IMAGE IS: THIS ISN'T GIVEN, BUT WON'T MATTER

$$f(x,y) = \delta\left(x-x_0,\, y-\tfrac{L}{2}\right) + \delta\left(x-x_0,\, y+\tfrac{L}{2}\right)$$

IF K-SPACE, THIS BECOMES:

$$F(k_x,k_y) = e^{-j2\pi\left(k_x x_0 + k_y \frac{L}{2}\right)} + e^{-j2\pi\left(k_x x_0 - k_y \frac{L}{2}\right)}$$

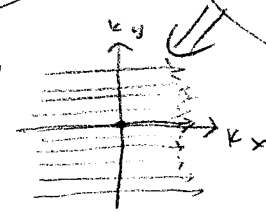
$$F(k_x,k_y) = e^{-j2\pi k_x x_0}\left(e^{-j2\pi k_y \frac{L}{2}} + e^{j2\pi k_y \frac{L}{2}}\right)$$

$$F(k_x,k_y) = e^{-j2\pi k_x x_0}\left[2\cos\left(2\pi \tfrac{L}{2} k_y\right)\right]$$

$$F(k_x,k_y) = 2e^{-j2\pi k_x x_0}\cos(\pi L k_y)$$

ANSWER 1:

IF WE SAMPLED THE CENTER OF K-SPACE IN k_y, OUR K-SPACE SAMPLES WILL NEVER ALL BE ZERO, SO WE CAN NEVER HAVE A ZERO IMAGE. THIS IS BECAUSE $\cos(\pi L k_y) = 1$ AT $k_y = 0$.



LIKE THIS, BUT THIS ISN'T WHAT I DREW!

SO:

(a) IMAGE WILL NEVER BE ZERO. ALWAYS SOME DELTA'S ALIASING IN.

(b) WE'LL GET A SINGLE IMPULSE WHENEVER WE SAMPLE THE $\cos(\pi L k_y)$ AT ONLY THE MAXIMA OR MINIMA

$$\pi L k_y = 0, \pm\pi, \pm 2\pi, \ldots$$

$$k_y = 0, \pm\tfrac{1}{L}, \pm\tfrac{2}{L}, \ldots$$

OR WHEN:

$$\Delta k_y = \frac{n}{L}, \quad n \text{ POSITIVE INTEGER}$$

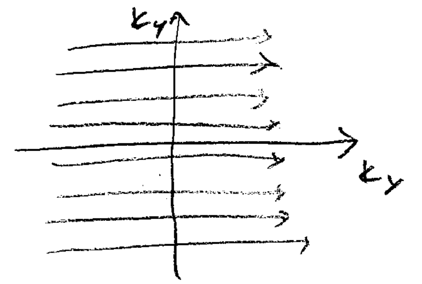
$$FOV_y = \frac{1}{\Delta k_y} = \frac{L}{n}, \quad n \text{ POSITIVE INTEGER} \Rightarrow$$

$FOV_y = \dfrac{L}{n}$, n POSITIVE INTEGER WILL GIVE A SINGLE DELTA IN IMAGE.

(c) NO. SHIFTING THINGS IN y WILL JUST INTRODUCE A LINEAR PHASE IN k_y, WHICH WILL IN TURN SIMPLY SHIFT THE LOCATION OF THE SINGLE DELTA (PART b) OR TWO DELTAS (PART a).

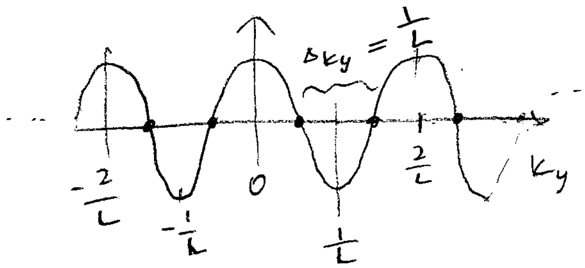
ANSWER 2:

AS DRAWN, WE <u>ARE NOT</u> SAMPLING THE CENTER OF k_y-SPACE!



IF THESE ARE SYMMETRIC ABOUT k_y, THEN:

(a) THE IMAGE WILL BE ZERO IF WE SAMPLE THE $\cos(\pi L k_y)$ AT ITS ZEROS!



SO, IF:

$$\Delta k_y = \frac{n}{L}, \quad n = 1, 3, 5, \ldots$$

WE HIT THE ZEROS!

$$FOV_y = \frac{1}{\Delta k_y} = \frac{L}{n}, \quad n = 1, 3, 5, \ldots$$

$\boxed{\text{ZERO IMAGE WHEN } FOV_y = \frac{L}{n},\ n = 1, 3, 5, \ldots}$

(b) SINGLE IMPULSE WHEN SAMPLING AT MAXIMA OR MINIMA ONLY:

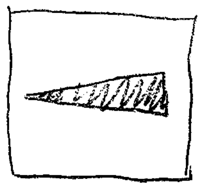
$\Delta k_y = \frac{n}{L}, \quad n = 2, 4, 6, \ldots \qquad FOV_y = \frac{L}{n}, \quad n = 2, 4, 6, \ldots$

$\boxed{\text{SINGLE DELTA WHEN } FOV_y = \frac{L}{n},\ n = 2, 4, 6, \ldots}$

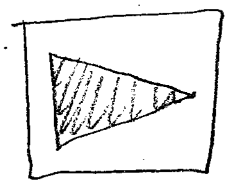
(c) NO, FOR THE SAME REASON AS IN ANSWER (1).

PROBLEM 2:

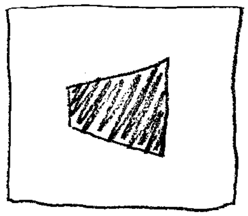
(a) IF $G_{y\,max}$ IS CUT IN HALF, WE DOUBLE FOV_y (AND CONSEQUENTLY GET HALF THE RESOLUTION IN y):



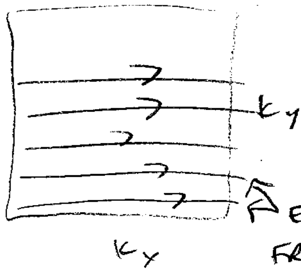
(b) INVERTING G_x GRADIENT FLIPS K-SPACE AROUND THE k_y AXIS, WHICH FLIPS THE IMAGE AROUND THE y AXIS:



(c) CUTTING THE LOW-PASS FILTER BANDWIDTH IN HALF WHILE LEAVING EVERYTHING ELSE CONSTANT WILL SIMPLY "FILTER OUT" AN ADDITIONAL HALF OF OUR IMAGE IN THE x DIRECTION!

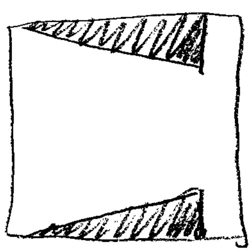


(d) IN K-SPACE, THIS PUTS A LINEAR PHASE IN THE k_y DIRECTION

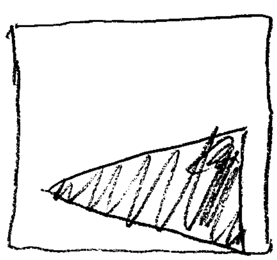


EACH LINE IN K-SPACE IS 180° OUT OF PHASE FROM ADJACENT LINES.

A LINEAR PHASE IN k_y IS A SHIFT IN y (PROPERTY OF F.T.). IN THIS CASE, A PHASE INCREMENT OF 180° BETWEEN EACH LINE OF K-SPACE CORRESPONDS TO A $\frac{1}{2}$ FOV SHIFT. (TRY IT IN MATLAB!)



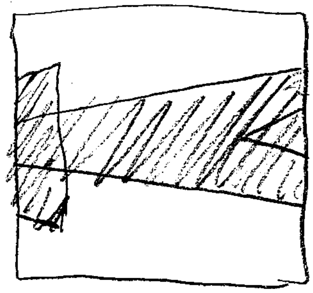
e) THIS CORRESPONDS TO A LINEAR PHASE IN THE k_y DIRECTION, BUT W/ AN INCREMENT OF 90° BETWEEN EACH LINE IN K-SPACE, CORRESPONDING TO A $\frac{1}{4}$ FOV SHIFT. (TRY IT!)



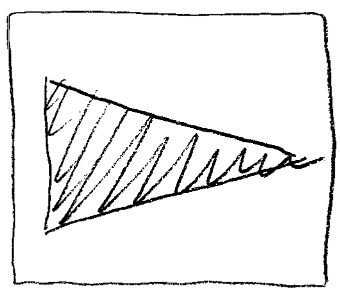
f) IF G_x AMPLITUDE IS DOUBLED, WE:

- CUT FOV_x IN HALF
- DOUBLE RESOLUTION IN x DIRECTION

HOWEVER, WE DIDN'T UPDATE OUR ANTI-ALIASING FILTER BW, SO WE ARE GOING TO GET ALIASING IN X DIRECTION!



g) THIS WILL HAVE 2 EFFECTS. FIRST, OUR IMAGE SCALING WILL BE DIFFERENT, BUT WE WON'T SEE THAT. SECOND, THE $e^{j2\pi(k_x x + k_y y)}$ INSTEAD OF $e^{-j2\pi(k_x x + k_y y)}$ WILL FLIP OUR IMAGE IN BOTH DIRECTIONS (X AND Y).



WE ONLY SEE THE FLIP IN X BECAUSE OUR OBJECT IS SYMMETRIC IN Y!

PROBLEM 3:

THE OBJECT POSITION IN THE y DIRECTION IS GIVEN BY:

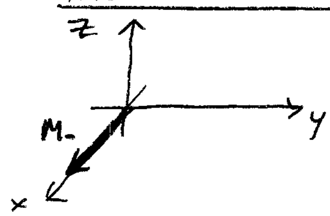
$$y(t) = y_o + vt$$

IF A y GRADIENT $G_y(t)$ IS ON, THE OBJECT HAS FREQUENCY $f(t)$ THAT DEPENDS ON THE y POSITION:

$$f(t) = \frac{\gamma}{2\pi} G_y(t)(y_o + vt) \Leftarrow \boxed{\text{IN THE ROTATING FRAME!}}$$

RIGHT AFTER THE FIRST 90°_y TIP, OUR MAGNETIZATION IS ALL ALONG THE X-AXIS, AS SHOWN. WE'LL DEFINE THAT AS ZERO PHASE:

AFTER FIRST TIP:



a) NOW WE WANT TO FIND THE MAGNITUDE & PHASE JUST PRIOR TO THE SECOND EXCITATION.

MAGNITUDE WILL BE M_o, SINCE WE ARE IGNORING T_1, T_2 RELAXATION!

BUT WHAT IS PHASE? (AT TIME $t = T$)

$$\underset{\substack{\Uparrow \\ \text{PHASE AT} \\ \text{TIME } t}}{\phi(t)} = \underset{\substack{\uparrow \\ \text{PUT IT IN} \\ \text{RADIANS!}}}{2\pi} \int_0^t f(\tau)\, d\tau \Leftarrow \boxed{\text{WE FIND PHASE BY INTEGRATING FREQUENCY OVER TIME, AS WE HAVE DONE IN CLASS SEVERAL TIMES.}}$$

SO:

$$\phi(t=T) = 2\pi \int_0^{\frac{T}{2}} \frac{\gamma}{2\pi} \underset{\substack{\uparrow \\ G_y(t) = -G \text{ FROM } t=0 \\ \text{TO } t = \frac{T}{2}}}{(-G)} (y_o + v\tau)\, d\tau + 2\pi \int_{\frac{T}{2}}^{T} \frac{\gamma}{2\pi} \underset{\substack{\uparrow \\ G_y(t) = +G \text{ FROM} \\ t = \frac{T}{2} \\ \text{TO } t = T}}{(G)} (y_o + v\tau)\, d\tau$$

$$\phi(t=T) = -G\gamma\left[y_0\tau + \tfrac{1}{2}v\tau^2\right]_0^{\frac{T}{2}} + G\gamma\left[y_0\tau + \tfrac{1}{2}v\tau^2\right]_{\frac{T}{2}}^{T}$$

FYI

$$\phi(t=T) = -\gamma G\left[y_0\frac{T}{2} + \frac{1}{2}v\frac{T^2}{4}\right] + \gamma G\left[y_0 T + \frac{1}{2}vT^2 - y_0\frac{T}{2} - \frac{1}{2}v\frac{T^2}{4}\right]$$

$$\phi(t=T) = \gamma G\left[\cancel{-y_0\frac{T}{2}} - \frac{1}{2}v\frac{T^2}{4} + \cancel{y_0 T} - \cancel{y_0\frac{T}{2}} + \frac{1}{2}vT^2 - \frac{1}{2}v\frac{T^2}{4}\right]$$

$$\phi(t=T) = \gamma G\left(-v\frac{T^2}{4} + \frac{1}{2}vT^2\right) = \underbrace{\gamma\left(\frac{GT^2}{4}\right)}_{\text{CONSTANT}} v$$

PHASE IS PROPORTIONAL TO V!

THIS IS CALLED "VELOCITY ENCODING".

INSTEAD OF ENCODING y POSITION IN PHASE LIKE WE DO w/ NORMAL PHASE ENCODING, WE ENCODE y VELOCITY IN PHASE!

SO:

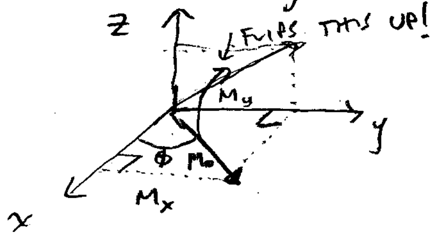
$$\boxed{\begin{aligned} |M_{xy}| &= M_0 \\ \angle M_{xy} &= \gamma\frac{GT^2}{4}v \end{aligned}}$$

AT TIME $t = T$ IMMEDIATELY BEFORE SECOND TIP.

$\phi(t=T)$

IN RADIANS

b) OUR SECOND TIP IS A -90°_y, OR A -90° TIP AROUND THE y AXIS. CONCEPTUALLY, THIS FLIPS M_x TO M_z BUT LEAVES M_y ALONG y AXIS (NEGATIVE OR POSITIVE, DEPENDING ON PHASE).



SO AFTER OUR SECOND TIP, WE'LL HAVE M_{xy} IN THE y DIRECTION (PHASE = 90° OR 270°) W/ MAGNITUDE:

$$|M_{xy}| = |M_0 \sin(\phi)|$$

$$\phi = \gamma \cdot \frac{GT^2}{4} \cdot v$$

SO:

$$\boxed{\begin{aligned} |M_{xy}| &= \left|M_0 \sin\left(\gamma \frac{GT^2}{4} v\right)\right| \\ \angle M_{xy} &= \begin{cases} \frac{\pi}{2}, & \sin\left(\gamma \frac{GT^2}{4} v\right) > 0 \\ \frac{3\pi}{2}, & \sin\left(\gamma \frac{GT^2}{4} v\right) < 0 \end{cases} \end{aligned}}$$

c) WE GET ZERO SIGNAL WHEN:

$$\sin\left(\gamma \frac{GT^2}{4} v\right) = 0$$

THE $\gamma \frac{GT^2}{2} v = 0$ CASE IS ZERO VELOCITY...

OR: $\gamma \frac{GT^2}{4} v = \pi$

SOLVING FOR V:

$$\gamma \frac{GT^2}{4} v = \pi \qquad \boxed{v = \frac{4\pi}{\gamma G T^2}}$$

d) $G = 0.3\ G/cm$, $T = 4\ ms$, $v = 20\ cm/s$

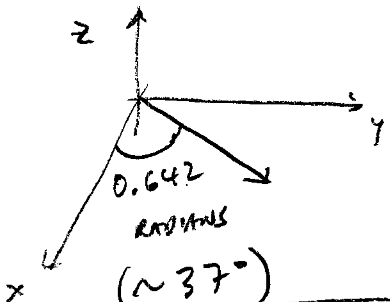
$$\frac{\gamma}{2\pi} = 42.58\ \frac{MHz}{T}$$

$$\gamma = 2\pi \cdot \left(42.58\ \frac{MHz}{T}\right)$$

WHAT IS THE PHASE OF M_{xy} IMMEDIATELY BEFORE SECOND TIP?

$$\angle M_{xy} = \gamma \frac{GT^2}{4} v = \frac{2\pi \left(42.58\times10^{6} \frac{1}{T\cdot s}\right)\left(0.3\times10^{-4} \frac{T}{cm}\right)\left(4\times10^{-3} s\right)^2 \left(20 \frac{cm}{s}\right)}{4}$$

$$\angle M_{xy} = 0.642 \text{ RADIANS} \approx 37^\circ$$



IN ORDER TO MAXIMIZE MR SIGNAL (M_{xy}) AFTER THE SECOND TIP, WE SHOULD PERFORM THE SECOND $-90°$ TIP AROUND AN AXIS ORIENTED AT $\boxed{\sim 37°}$ FROM THE x AXIS.

PROBLEM 4:

(a) HALF THE PHASE ENCODES CUTS TOTAL SCAN TIME IN HALF.

AND:

WE ONLY GET HALF THE RESOLUTION IN y DIRECTION,

SO: Δy IS DOUBLED:

WE KNOW:

$$SNR \propto (\Delta x)(\Delta y)(\Delta z)\sqrt{\text{TOTAL READOUT TIME}}\; f(\rho, T_1, T_2)$$

SO:

SNR INCREASES BY: $2\times$ FROM Δy DOUBLING

SNR DECREASES BY: $\frac{1}{\sqrt{2}}\times$ FROM TOTAL SCAN TIME REDUCTION

NET EFFECT:

$$\boxed{\text{SNR INCREASES TO } \frac{2}{\sqrt{2}} = \sqrt{2}}$$

- FOV IS UNCHANGED
- RESOLUTION IN y GETS WORSE BY A FACTOR OF 2
- SIGNAL BANDWIDTH IS UNCHANGED

(b) TOTAL SCAN TIME DOUBLES $\Rightarrow$ TOTAL READOUT TIME DOUBLES.

AND

- FOV DOUBLES IN Y DIRECTION.
- RESOLUTION IS UNCHANGED
- SIGNAL BANDWIDTH IS UNCHANGED

SNR INCREASES TO $\sqrt{2}$ FROM DOUBLING IN TOTAL READOUT TIME.

(c) – READOUT DURATION DOUBLES (SINCE WE GO OUT TWICE AS FAR IN k_x WITHOUT INCREASING GRADIENT AMPLITUDE.

- FOV_x AND FOV_y ARE CUT IN HALF
- RESOLUTION DOUBLES (GETS BETTER) IN BOTH x AND y

SNR $\Rightarrow$ $\begin{cases} \frac{1}{4} \times \text{ FROM VOXEL SIZE REDUCTION} \\ \sqrt{2} \times \text{ FROM DOUBLING READOUT TIME} \end{cases}$

$$\boxed{SNR = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}}$$

SIGNAL BANDWIDTH IS CUT IN HALF.