

- HW #6 WILL BE AVAILABLE THURSDAY
- ADVANCED TOPIC LECTURE RESCHEDULED
- MIDTERM WILL BE GRADED BY BEGINNING OF NEXT WEEK

LAST THURSDAY:

- LINEAR GRADIENT FIELDS $\begin{matrix} \nearrow G_x \\ \rightarrow G_y \\ \searrow G_z \end{matrix}$

$$B_z = B_0 + G_x x + G_y y + G_z z$$

↑ ALLOW US TO SPATIALLY VARY FREQUENCY, SINCE:

$$f = \frac{\gamma B_0}{2\pi}$$

- RF FIELD $\rightarrow \vec{B}_1$

↑ LINEARLY OR CIRCULARLY POLARIZED IN THE XY PLANE, USED TO "TIP" OR "EXCITE" SPINS

- MAIN ^{POLARIZATION} FIELD $\rightarrow \vec{B}_0$ $\vec{B}_0 = B_0 \hat{k}$

↑ ALWAYS IN Z DIRECTION.

- SAMPLING, OR READOUT, OR AQC \rightarrow WE PICK UP THE NMR SIGNAL IN A RADIOFREQUENCY COIL, AND SAMPLE IT OVER A SET PERIOD OF TIME.

- PULSE SEQUENCE DIAGRAM:

• MRI HAS TO DO WITH MANIPULATING \vec{M} OVER TIME TO GET THE SIGNAL WE DESIRE.

THE BLOCH EQUATION:

IN GENERAL, THE BEHAVIOR OF THE MAGNETIZATION VECTOR \vec{M} OF A SPIN SYSTEM (SAY A GIVEN VOXEL) IS DESCRIBED BY THE BLOCH EQUATION:

$$\frac{d\vec{M}}{dt} = \underbrace{\vec{M} \times \gamma \vec{B}}_{\text{PRECESSION}} - \underbrace{\frac{M_x \hat{i} + M_y \hat{j}}{T_2}}_{T_2 \text{ DELAY}} - \underbrace{\frac{(M_z - M_0) \hat{k}}{T_1}}_{T_1 \text{ RECOVERY}}$$

- WE WILL SOLVE THIS IN A MOMENT TO DEVELOP A MATHEMATICAL DESCRIPTION OF MR.

- THIS IS A PHENOMENOLOGICAL EQN. ! YOU CANNOT DERIVE FROM FIRST PRINCIPLES!

PRECESSION:

- AT THERMAL EQUILIBRIUM, \vec{M} AND \vec{B} WILL POINT IN THE SAME DIRECTION, SO:

$$\vec{M} \times \gamma \vec{B} = 0$$

- IF \vec{M} IS MADE TO POINT IN A DIFFERENT DIRECTION THAN \vec{B} , PRECESSIONAL BEHAVIOR OF THE MAGNETIZATION WILL OCCUR.

- FROM CLASSICAL MECHANICS, THE TORQUE APPLIED TO A DIPOLE MOMENT \vec{u} IN THE PRESENCE OF \vec{B} IS:

$$\text{TORQUE} = \vec{u} \times \vec{B}$$



RATE OF CHANGE OF ANGULAR MOMENTUM VECTOR $\vec{\Phi}$

$$\frac{d\vec{\Phi}}{dt} = \vec{u} \times \vec{B}$$

RECALL THAT $\vec{u} = \gamma \vec{\Phi}$, SO WE CAN WRITE:

↑
DIPOLE
(MAGNETIC
MOMENT)

$$\gamma \frac{d\vec{\Phi}}{dt} = \gamma \vec{u} \times \vec{B}$$

OR:

$$\frac{d\vec{u}}{dt} = \vec{u} \times \gamma \vec{B}$$

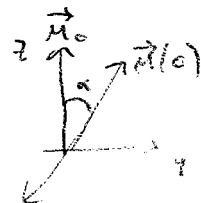
SUMMING EACH DIPOLE OVER THE SPIN SYSTEM ($\vec{M} = \sum_{n=1}^{N/2} \vec{u}_n$):

$$\boxed{\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}}$$

← FIRST TERM IN BLOCK EQUATION ⇒ PRECESSION!

ASSUME $\vec{B}(t) = B_0 \hat{k}$ (AFTER EXCITATION) AND THAT WE'VE STATIC IN Z DIRECTION

"TIPPED" \vec{M} AWAY FROM \vec{M}_0 BY ANGLE α :



SOLUTION IS:

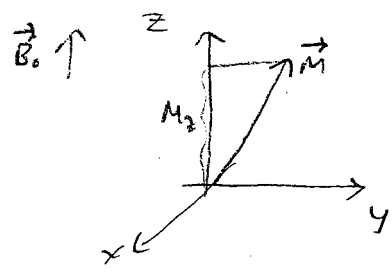
$$\vec{M}(t) = M_0 \sin \alpha [\sin(-\gamma B_0 t) \hat{i} + \cos(-\gamma B_0 t) \hat{j}] + M_0 \cos \alpha \hat{k}$$

FREQUENCY IS $\frac{\gamma}{2\pi} B_0$ PRECESSION!

RELAXATION: ← WE'RE ONLY GOING TO CONSIDER MAGNETIC DIPOLE-DIPOLE INTERACTIONS.

- NOW THAT WE'VE DESCRIBED PRECESSION, LET'S LOOK AT THE OTHER TERMS OF THE BLOCH EDN.

- FIRST, SOME DEFINITIONS:



$M_z =$ LONGITUDINAL MAGNETIZATION

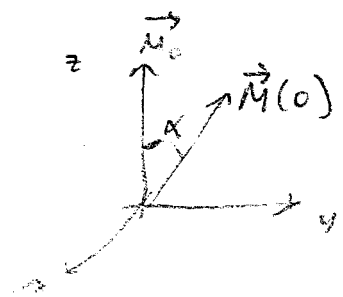
$M_{xy} =$ TRANSVERSE MAGNETIZATION

LONGITUDINAL RELAXATION:

FROM BLOCH EDN: (IGNORING PRECESSION!)

$$\frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1}$$

AFTER A TIP BY ANGLE α :



SOLUTION IS:

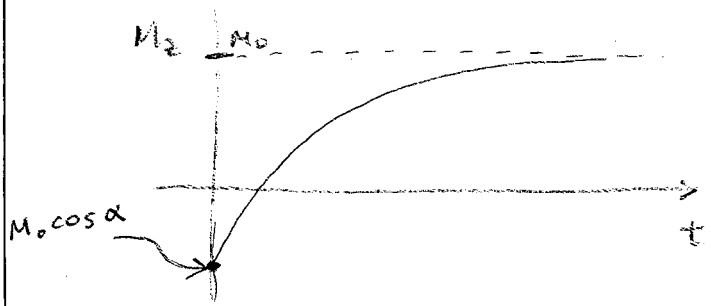
$$M_z = M_0 + \underbrace{[M_0 \cos \alpha - M_0]}_{M_z(0)} e^{-t/T_1}$$

FOLLOWING AN $\alpha = 90^\circ$ TIP:

$$M_z = M_0 - M_0 e^{-t/T_1} = M_0 (1 - e^{-t/T_1})$$

FOLLOWING AN $\alpha = 180^\circ$ TIP:

$$M_z = M_0 + [-M_0 - M_0] e^{-t/T_1} = M_0 - 2M_0 e^{-t/T_1}$$



$T_1 \Rightarrow$ "SPIN-LATTICE" TIME CONSTANT

CHARACTERIZES RETURN TO EQUILIBRIUM IN Z DIRECTION

TRANSVERSE RELAXATION:

$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$

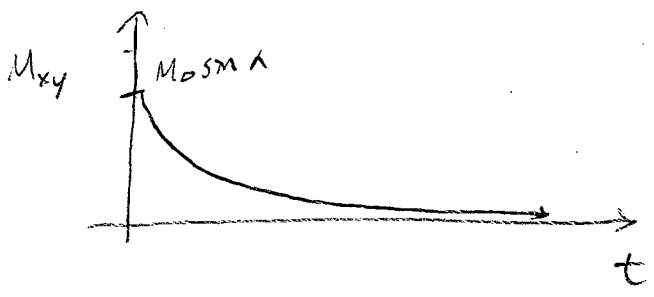
FROM BLOCH EQN, IGNORING PRECESSION!

AFTER A TIP BY α :

$$M_{xy} = M_0 \sin \alpha e^{-t/T_2}$$

- REMEMBER THAT OUR MR SIGNAL IS PROPORTIONAL TO M_{xy} (SINCE OUR RF COIL IS ORIENTED SO IT IS SENSITIVE IN THE XY PLANE, AND PRECESSION OCCURS AT OUR RF FREQUENCY IN THE XY PLANE ABOUT THE Z AXIS!)

SO: T₂ IS A SIMPLE EXPONENTIAL SIGNAL DECAY!



T₂ => "SPIN-SPIN" TIME CONSTANT

CHARACTERIZES DELAY OF TRANSVERSE MAGNETIZATION.

T₁ INCREASES W/ FIELD STRENGTH!

SOME BIOLOGICAL TISSUES:

	T ₂ (ms)	T ₁ (AT 1.5 T) (ms)
GRAY MATTER	100	920
WHITE MATTER	92	780
MUSCLE	47	870
FAT	85	270
KIDNEY	58	650
LIVER	43	495

CHEMICAL SHIFT:

- SMALL DISPLACEMENT OF RESONANT FREQUENCY DUE TO SHIELDING CREATED BY THE ORBITAL MOTION OF SURROUNDING ELECTRONS IN RESPONSE TO \vec{B}_0 .

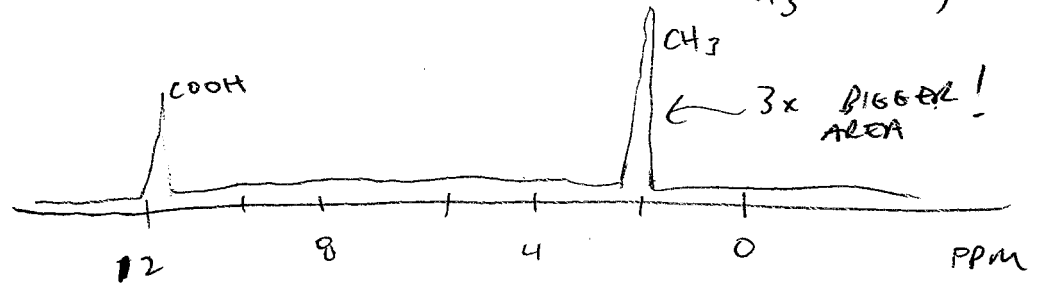
FOR ANY GIVEN NUCLEUS:

$$B_{eff} = B_0(1 - \sigma)$$

↑
SHIELDING CONSTANT
SPECIFIC TO CHEMICAL ENVIRONMENT OF NUCLEUS.

- BY CONVENTION IN NMR, THE FREQUENCY AXIS IS REVERSED.
- WE OFTEN GIVE SHIFTS IN PARTS PER MILLION (PPM)

NMR SPECTRUM OF ACETIC ACID (CH_3COOH)



- IN HUMANS, THE MOST ABUNDANT 1H SPECIES ARE WATER (H_2O) AND FAT (LIPIDS \Rightarrow MANY CH_2 GROUPS)

FAT IS \approx 3.5 ppm LOWER THAN H_2O .