ANNOUNCEMENTS:

- MIDTERMS BACK ⇒ COME TO OFFICE HOURS RIGHT AFTER
- FEEDBACK ⇒ THANK YOU!
- TOPIC ⇒ MORE MR, LESS ULTRASOUND?
- FINAL PROJECT PROPOSALS

## MORE ON THE FOURIER INTERPRETATION OF THE SIGNAL EQN.

LAST WEEK, WE DERIVED THE MR SIGNAL EQUATION:

$$s(t) = \int_x \int_y m(x,y)\, e^{-j2\pi[k_x(t)x + k_y(t)y]}\, dx\, dy$$

WHERE: $$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\, d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\, d\tau$$

WE MENTIONED THAT:

$$s(t) = M(k_x(t), k_y(t)) \quad \text{WHERE:}$$

$M(k_x, k_y)$ IS THE 2D FT OF $m(x,y)$.

MR IMAGE ACQUISITION CAN BE THOUGHT OF AS TRAVERSING K-SPACE USING OUR GRADIENTS $G_x(t)$ AND $G_y(t)$, GATHERING SAMPLES OF $M(k_x, k_y)$ FROM OUR TIME SIGNAL $s(t)$.

HOW WE TRAVERSE K-SPACE IS ENTIRELY A FUNCTION OF WHAT WE DO WITH OUR GRADIENTS OVER TIME. OUR POSITION IN K-SPACE AT ANY POINT IN TIME IS SIMPLY A FUNCTION OF THE TIME INTEGRAL OF OUR GRADIENTS!

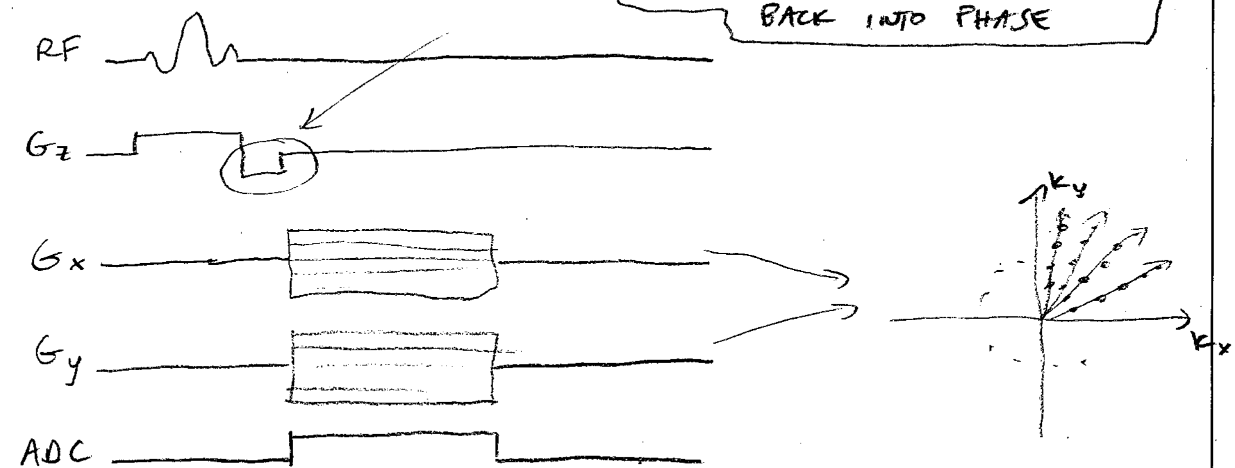
DESIGNING THESE GRADIENT TRAJECTORIES TO GIVE US A FULL SET OF SAMPLES OF $M(k_x, k_y)$ IS A FUNDAMENTAL PROBLEM IN MRI PULSE SEQUENCE DESIGN.

LET'S TAKE A LOOK AT SOME COMMON K-SPACE TRAJECTORIES.

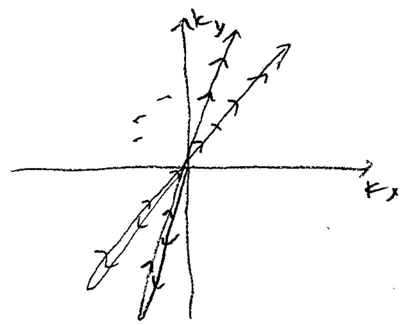
## 2D PROJECTION RECONSTRUCTION

- SIMILAR TO X-RAY CT

EXPLAIN WHY WE NEED THIS: IT BRINGS THE WHOLE SLICE BACK INTO PHASE




ALTERNATELY, WE COULD DO THE FOLLOWING:




THIS IS A <u>NON-CARTESIAN</u> TRAJECTORY => REQUIRES EITHER <u>GRIDDING</u> OR <u>BACKPROJECTION</u> FOR RECON!

FOR HALF AS MANY REPETITIONS.

WHAT WOULD MY GRADIENTS NEED TO LOOK LIKE?

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THEORETICALLY, IF $m(x,y)$ IS A REAL FUNCTION, THEN $M(k_x,k_y)$ IS <u>HERMITIAN</u>. THAT IS:

$$M(-k_x,-k_y) = M^*(k_x,k_y)$$

THUS, QUADRANT I CAN BE USED TO FIND QUADRANT III, AND II TO FIND IV.

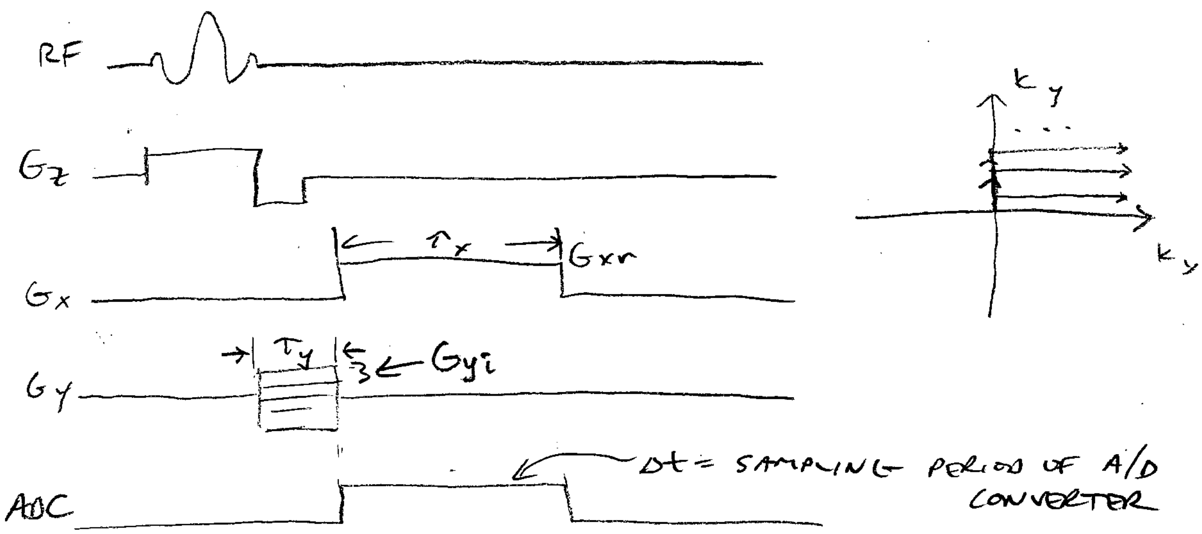
IN PRACTICE, HOWEVER, $m(x,y)$ IS USUALLY NOT REAL VALUED (DUE TO PHASE SHIFTS FROM A VARIETY OF SOURCES) AND THE HERMITIAN ASSUMPTION BREAKS DOWN.

HOWEVER, WE CAN GET AWAY WITH SOME REDUCTIONS IN SAMPLING.

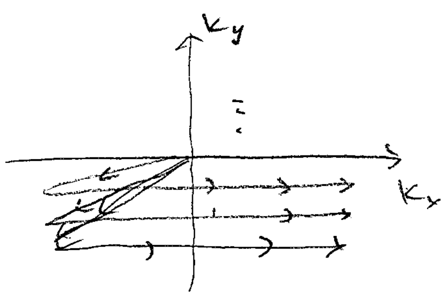
TALK ABOUT PARTIAL PHASE, COMPRESSED SENSING, ETC.

## 2D FT (Fourier Transform)

- CARTESIAN TRAJECTORY



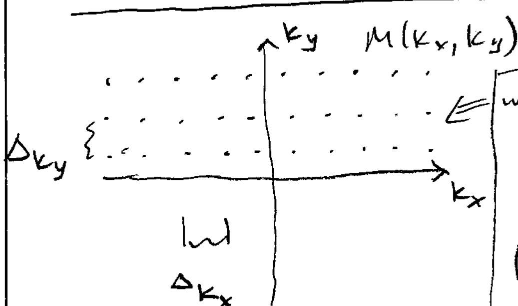

ALTERNATELY, WE TYPICALLY DO THIS:




SHOW SLIDES FOR PHASE ENCODING AND FREQUENCY ENCODING

## SAMPLING REQUIREMENTS IN 2D FT IMAGING

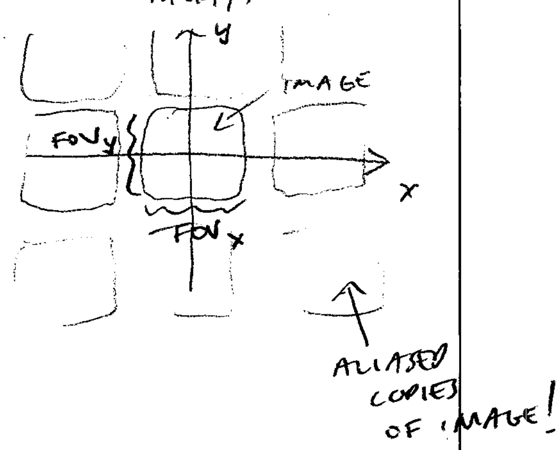
### FIELD OF VIEW (FOV):




WE ARE SAMPLING $M(k_x, k_y)$ (IN FOURIER DOMAIN!)

$\Longleftrightarrow$

IN SPATIAL DOMAIN $m(x, y)$




$$\boxed{FOV_x = \frac{1}{\Delta k_x} = \text{SAMPLING RATE ALONG } k_x}$$

$$\boxed{FOV_y = \frac{1}{\Delta k_y} = \text{SAMPLING RATE ALONG } k_y}$$

IN THE SPECIFIC CASE OF 2DFT IMAGING:

$$\boxed{FOV_x = \frac{1}{\Delta k_x} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t} \qquad FOV_y = \frac{1}{\Delta k_y} = \frac{1}{\frac{\gamma}{2\pi} G_{yi} \tau_y}}$$

$\Delta t$ ← A/D SAMPLING RATE

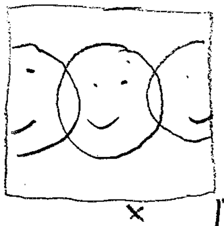
$G_{xr}$ ← READOUT GRADIENT AMPLITUDE

$\tau_y$ ← LENGTH OF TIME PHASE ENCODING GRADIENT IS ON

$G_{yi}$ ← INCREMENTAL GRADIENT AMPLITUDE BETWEEN PHASE ENCODES.

WHAT IF WE UNDER SAMPLE?

SHOW ALIASED IMAGE FROM POWER POINT.

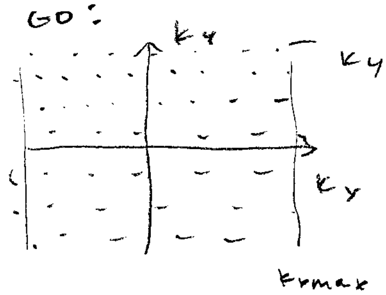



← UNDERSAMPLED BY A FACTOR OF 2 IN X-DIRECTION

- WHAT IF WE APPLY ANTI-ALIASING FILTER IN $k_x$?
- CAN WE DO THAT IN $k_y$?

SPATIAL RESOLUTION:

- SPATIAL RESOLUTION CORRESPONDS TO HOW FAR OUT IN K-SPACE WE GO:



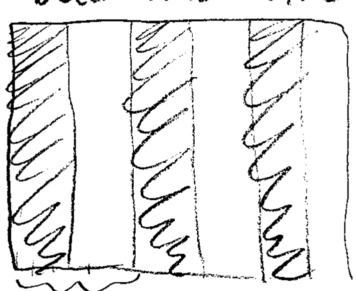

$$\boxed{\delta_x = \frac{1}{2k_{xmax}} = \frac{1}{\frac{\gamma}{2\pi} G_{xr} T_x} \qquad \delta_y = \frac{1}{2k_{ymax}} = \frac{1}{\frac{\gamma}{2\pi} 2G_{yp} T_y}}$$

$T_x$ ← TOTAL A/D TIME

$G_{yp}$ ← MAX PHASE ENCODE AMPLITUDE

- RESOLUTION IS THE HALF-CYCLE WIDTH OF THE HIGHEST SPATIAL FREQUENCY RECORDED IN EACH DIRECTION!

WHY DOES THIS MAKE SENSE?




HALF CYCLE WIDTH IS PIXEL WIDTH!

EXAMPLE:

WE WANT: $FOV_x = FOV_y = 25.6$ cm.

AND: $\delta_x = \delta_y = 0.1$ cm (1 mm)

⇒ THAT MEANS OUR K-SPACE DATA MATRIX WILL BE 256×256 POINTS.

SAMPLING RATE?

IF $G_{xr} = 0.3 \; \frac{G}{cm}$, WHAT IS $\Delta t$?

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \Delta t}$$

$$\boxed{\begin{gathered}\Delta t \Rightarrow 30.58 \; \mu s \\ \frac{1}{\Delta t} \Rightarrow 32.7 \; kHz\end{gathered}}$$

READOUT TIME

WHAT IS $\tau_x$?

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr} \tau_x}$$

$$\boxed{\tau_x = 7.83 \; ms}$$

IS THIS OKAY FROM A $T_2$ DECAY PERSPECTIVE??

IF $\tau_y = 4$ ms, WHAT IS $G_{yi}$?

$$\boxed{G_{yi} = 2.3 \; mG/cm}$$

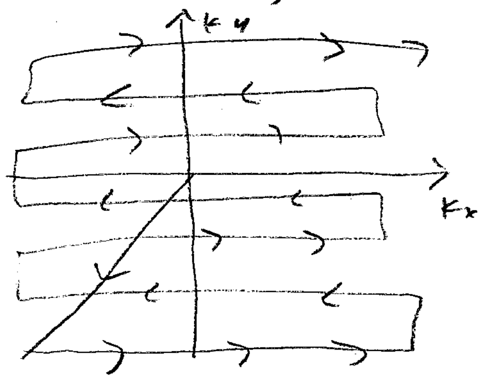
WHAT IS $G_{yp}$?

$$\boxed{G_{yp} = 0.29 \; \frac{G}{cm}}$$

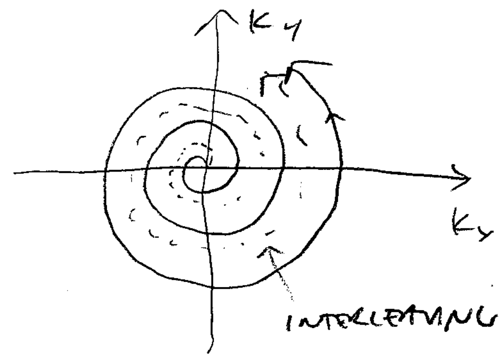
# ECHO PLANAR IMAGING & SPIRALS:

- SHOW DIAGRAMS FROM PHYSICS SLIDES

ECHO PLANAR IMAGING (EPI)




SPIRAL




WHAT DO GRADIENTS LOOK LIKE??

## EXTENDING TO 3D

- CAN FORMULATE SIGNAL EQUATION ACROSS 3 DIMENSIONS
- IMAGING $m(x,y,z)$
- THEN WE HAVE:

$$s(t) = \int_x \int_y \int_z m(x,y,z)\, e^{-j2\pi[k_x(t)x + k_y(t)y + k_z(t)z]}\, dx\,dy\,dz$$

WHERE: $$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau$$

$$k_z(t) = \frac{\gamma}{2\pi}\int_0^t G_z(\tau)\,d\tau$$

SO:

$$s(t) = M\left(k_x(t), k_y(t), k_z(t)\right) \Leftarrow \text{FOURIER INTERPRETATION}$$

## 3D FT IMAGING:


$\boxed{\text{IMAGING TIME:}}$ IN 2D, IF I WANT A 256×256 IMAGE, I REPEAT ACQUISITION 256 TIMES (E.G., 256 PHASE ENCODES)

WHAT IF I WANT A 256×256×64 IMAGE?