- FERPA
- COURSE WEBSITE NOW UP - REVIEW ORIENTATIONS
- HOMEWORK => DUE TUESDAYS? ⇐ 1 WEEK MORE ON REVIEW. WILL UPDATE SYLLABUS AND DUE DATES.

SIGNALS & SYSTEMS REVIEW:

SIGNAL: MATHEMATICAL FUNCTION OF ONE OR MORE INDEPENDENT VARIABLES, USED TO MODEL PHYSICAL PROPERTIES OR PROCESSES OF INTEREST.

- CONTINUOUS ← DISTRIBUTION OF X-RAY ATTENUATION
- DISCRETE ← SAMPLED CONTINUOUS SIGNALS
- MIXED ← CT SIGNAL, WITH CONTINUOUS DISTRIBUTION OF X-ray ATTENUATION AND MULTIPLE ACQUISITIONS AT DISCRETE ANGLES.

SYSTEM: RESPONDS TO SIGNALS BY PRODUCING NEW SIGNALS

NOTATION:

$f(x,y)$, $-\infty \le x, y \le \infty$ ∈ CONTINUOUS SIGNAL

↑ 2 IND. VARIABLES!

2D IMAGES => PIXEL
3D IMAGES => VOXEL

SPECIAL SIGNALS:

(1) POINT IMPULSE: → 1-D:

$$\delta(x) = 0, \quad x \neq 0$$
$$\int_{-\infty}^{\infty} f(x)\delta(x)\,dx = f(0)$$

$$\delta(x,y) = 0, \quad (x,y) \neq (0,0)$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x,y)\,dx\,dy = f(0,0)$$

- 2D POINT IMPULSE
- 2D IMPULSE FUNCTION
- 2D DELTA FUNCTION
- 2D DIRAC DELTA

↑ "PICKS OFF" THE VALUE OF $f(x,y)$ AT $f(0,0)$ UNDER INTEGRATION. "SIFTING PROPERTY"

WHY IS THIS IMPORTANT?

USED TO CHARACTERIZE IMAGE RESOLUTION! (A SMALL SOURCE THAT APPEARS LARGE OR BLURRED MEANS POOR RESOLUTION) = "POINT SPREAD FUNCTION"

SHIFTED 2D IMPULSE:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x-x_0, y-y_0)\,dx\,dy = f(x_0, y_0)$$

PROPERTIES OF POINT IMPULSE:

- SIFTING
- SCALING $\Rightarrow \delta(ax, by) = \frac{1}{|ab|}\delta(x,y)$
- EVEN $\Rightarrow \delta(-x,-y) = \delta(x,y)$

(2) LINE IMPULSE:

SOMETIMES USEFUL FOR CALIBRATING A MEDICAL IMAGING SYSTEM.

(3) COMB AND SAMPLING FUNCTIONS:

- 2D DELTA PICKS OFF THE VALUE OF A 2D SIGNAL AT A SINGLE POINT.
- IN MEDICAL IMAGING, WE OFTEN NEED TO PICK OFF VALUES ON A GRID OR MATRIX OF POINTS $\Rightarrow$ "SAMPLING"

1-D: $\text{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$ ⟸ - COMB OR "SHAH" FUNCTIONS

2-D: $\text{comb}(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \delta(x-m, y-n)$ - TEETH SPACED 1 APART

SAMPLING FUNCTION:

$$\delta_s(x,y;\underbrace{\Delta x;\Delta y}_{\text{THESE ARE JUST SPACING!}}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \delta(x-m\Delta x, y-n\Delta y)$$

(4) RECT FUNCTION:

FINITE ENERGY, UNIT TOTAL ENERGY

$$\text{rect}(x,y) = \begin{cases} 1, & |x| < \frac{1}{2} \text{ AND } |y| < \frac{1}{2} \\ 0, & |x| > \frac{1}{2} \text{ OR } |y| > \frac{1}{2} \end{cases}$$

VALUE AT $x, y = \pm\frac{1}{2}$ IS IMMATERIAL. $\frac{1}{2}$ IF YOU LIKE.

WHAT DOES THIS DO?

$$f(x,y)\,\text{rect}\left(x-x_0, y-y_0\right) \; ?$$

$$f(x,y)\,\text{rect}\left(\frac{x-x_0}{X}, \frac{y-y_0}{Y}\right) \; ?$$

CAN BE WRITTEN AS:

$$\text{rect}(x,y) = \text{rect}(x)\,\text{rect}(y)$$

"SEPARABLE"!

(5) SINC FUNCTION:

$$\text{sinc}(x,y) = \begin{cases} 1, & x=y=0 \\ \dfrac{\text{sin}(\pi x)\,\text{sin}(\pi y)}{\pi^2 x y}, & \text{else} \end{cases}$$

FINITE ENERGY, UNIT TOTAL ENERGY

$$\text{sinc}(x,y) = \text{sinc}(x)\,\text{sinc}(y) \Leftarrow \text{SEPARABLE!}$$

SEPARABLE SIGNALS:

$f(x,y)$ IS SEPARABLE IF

$$f(x,y) = f_1(x)\,f_2(y)$$

(6) EXPONENTIAL & SINUSOIDAL SIGNALS:

$$e(x,y) = e^{j2\pi(u_0 x + v_0 y)}$$ ← 2D COMPLEX EXPONENTIAL SIGNAL

SEPARABLE?

ALSO k_x, k_y.

"SPATIAL FREQUENCIES"

OR "FUNDAMENTAL FREQUENCIES"

RELATE TO TIME SIGNAL: $e^{j2\pi f_0 t}$

OF COURSE WE COULD HAVE SINUSOIDAL COMPONENTS:

$$s(x,y) = \sin[2\pi(u_0 x + v_0 y)]$$

$$c(x,y) = \cos[2\pi(u_0 x + v_0 y)]$$

AND:

$$e(x,y) = c(x,y) + j\,s(x,y)$$

WHAT DOES IT LOOK LIKE IF WE PLOT THE MAGNITUDE OF $e(x,y)$ AS AN IMAGE? P-23!

PERIODIC SIGNALS:

$f(x,y)$ IS PERIODIC IF THERE EXIST TWO POSITIVE CONSTANTS X, Y SUCH THAT:

$$f(x,y) = f(x+X, y) = f(x, y+Y) = f(x+X, y+Y)$$

WHAT IS THE PERIOD OF THE EXPONENTIAL FUNCTION

$$e(x,y) = e^{j2\pi(u_0 x + v_0 y)}$$?

$$X = \frac{1}{u_0} \qquad Y = \frac{1}{v_0}$$!