- HW7 DUE TUESDAY 11/17
- HW HELP SESSION THIS THURSDAY 5-6 pm
- SEE ME IF YOU NEED EXTRA TIME ON PROPOSAL
- ADVANCED TOPIC LECTURE, "COMPRESSED SENSING", FRIDAY 4-5 p.m.

LAST TIME:
- T_2^* DECAY
- SPIN ECHOES
- GRADIENT ECHOES
- THERMAL NOISE ("JOHNSON NOISE")

NOISE IN MRI (CONTINUED):

- IMAGE NOISE IN MRI IS A BI-VARIATE (COMPLEX-VALUED) ZERO-MEAN GAUSSIAN RANDOM PROCESS WITH REAL AND IMAGINARY COMPONENTS EACH POSSESSING VARIANCE σ_n^2.

- BECAUSE THE RECONSTRUCTED IMAGE IS NOT GENERALLY REAL VALUED, THE DISPLAYED IMAGE IS TYPICALLY THE ABSOLUTE VALUE OF THE COMPLEX IMAGE.
  - IN BACKGROUND REGIONS: GAUSSIAN ⇒ RAYLEIGH

$$\sigma^2 \Rightarrow \left(2 - \frac{\pi}{2}\right)\sigma_n^2$$

$$\mu \Rightarrow \sigma\sqrt{\pi/2}$$

  - IN REGIONS OF SIGNAL: GAUSSIAN ⇒ RICIAN
    - IF SNR IS HIGH, GAUSSIAN CAN APPROXIMATE RICIAN
    - NOT SO IF SNR IS LOW ⇒ NOISE THRESHOLDING THAT DEGRADES SNR

GETTING PRACTICAL ABOUT NOISE:

$$SNR \propto (\Delta x)(\Delta y)(\Delta z)\sqrt{\text{TOTAL READOUT TIME}}\; f(\rho, T_1, T_2)$$

$(\Delta x)(\Delta y)(\Delta z)$: VOXEL VOLUME

$f(\rho, T_1, T_2)$: CONTRAST (PROTON DENSITY, T_1, T_2)

EFFECT OF ACQUISITION TIME:

SIGNAL AVERAGING:
- SIGNALS ADD
- VARIANCES ADD

READOUT TIME:

ANTI-ALIASING FILTER BANDWIDTH
- NOISE VARIANCE PER SAMPLE IS PROPORTIONAL TO Δf

$$\sigma_n^2 \propto \Delta f = \frac{1}{\Delta t}$$

- DOUBLING THE SAMPLING PERIOD CUTS THE NOISE VARIANCE PER SAMPLE IN HALF, SO STANDARD DEVIATION IS REDUCED BY A FACTOR OF $\sqrt{2}$.

SO:

$$SNR \propto \sqrt{N_{AVERAGES} T_{READOUT}} = \sqrt{\text{TOTAL ACQUISITION TIME}}$$

$$= \sqrt{N_{AVERAGES} N_{PHASE\ ENCODES} T_{READOUT}}$$

EXAMPLE:

CONSIDER 2 2DFT SEQUENCES:

SEQ 1: 256 SAMPLES PER READOUT
SEQ 2: 512 SAMPLES PER READOUT

} GRADIENTS ARE THE SAME, SAME READOUT TIME

WHAT IS RELATIVE SNR?

SPATIAL RESOLUTION:

- VOXEL VOLUME IS DIRECTLY PROPORTIONAL TO SIGNAL,
- ARE WE JUST AS WELL OFF SCANNING AT TWICE THE RESOLUTION AND AVERAGING??

OTHER FACTORS:

$$f(\rho, T_1, T_2) \Rightarrow \text{CONTRAST!}$$

2D vs. 3D?

SPECTROSCOPIC IMAGING

- WE ORIGINALLY IGNORED CHEMICAL SHIFT IN SIGNAL EQN.

$$m(x,y) \longrightarrow m(x,y,f)$$

- FOR NON CHEMICAL SHIFT IMAGING:

$$I(x,y) = \int_f m(x,y,f)\, df$$

- IN SPECTROSCOPIC IMAGING, WE WANT:

$I(x,y,f)$ AS OUR IMAGE ← SPECTRUM AT EACH x, y POSITION

W/OUT CHEMICAL SHIFT:

$$s(t) = \int_x \int_y m(x,y)\, e^{-j2\pi k_x(t) x} e^{-j2\pi k_y(t) y}\, dx\, dy$$

W/ CHEMICAL SHIFT:

$$s(t) = \int_x \int_y \int_f m(x,y,f)\, e^{-j2\pi k_x(t) x} e^{-j2\pi k_y(t) y} e^{-j2\pi f t}\, df\, dx\, dy$$

COMPARE THIS W/ 3D FT: (OF $m(x,y,f)$)

$$M(k_x, k_y, k_f) = \int_x \int_y \int_f m(x,y,f)\, e^{-j2\pi (k_x x + k_y y + k_f f)}\, dx\, dy\, df$$

SO:

$$s(t) = M(k_x(t), k_y(t), k_f(t))$$

WHERE:

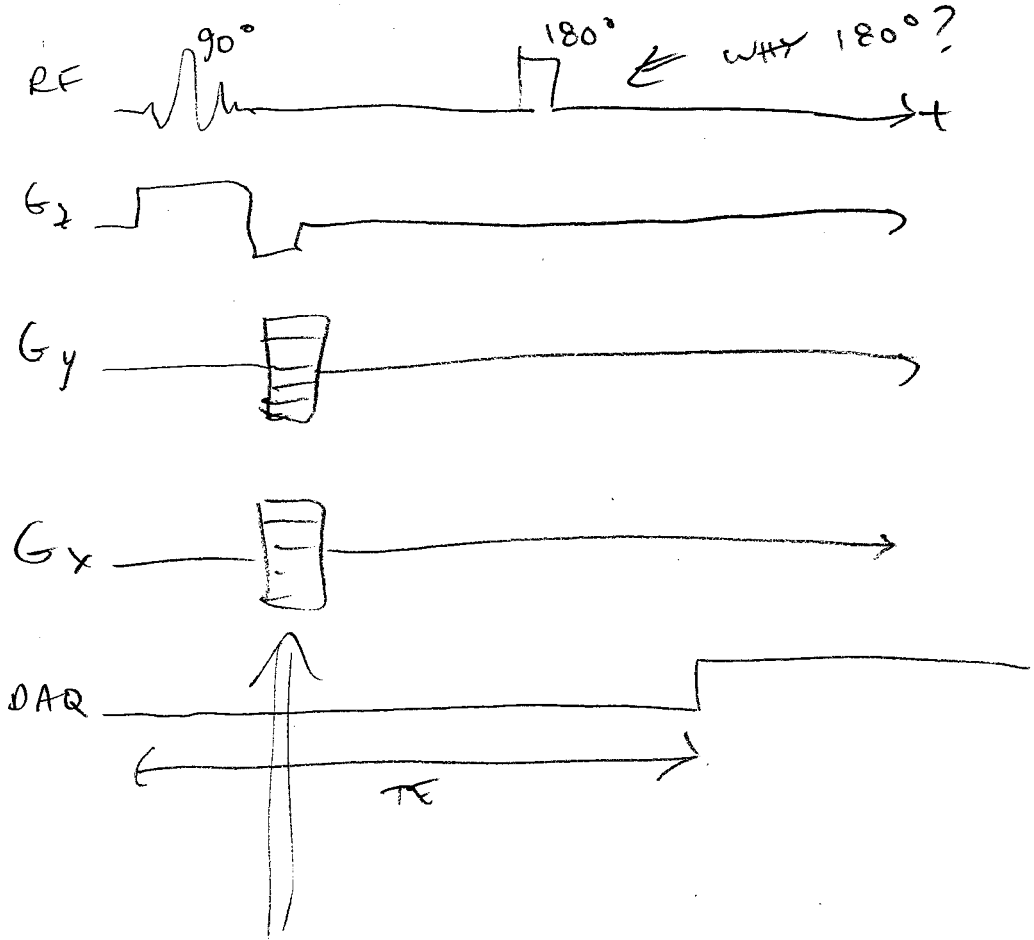
$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau)\, d\tau$$

$$k_f(t) = t$$

WE CAN TREAT THE CHEMICAL SHIFT AXIS f AS ANOTHER "SPATIAL" AXIS ALONG W/ x AND y!

WE MOVE THROUGH k_f JUST BY WAITING IN TIME!

TO FORM A 2D MRS IMAGE, WE REQUIRE FILLING OF $(k_x, k_y, k_f = t)$ SPACE!



PICK UP POINTS IN k_x, k_y ONE AT A TIME!