- HW 7 DUE TONIGHT — ADVANCED TOPICS: SENSE FRIDAY 4-5 PM
- A FEW WORDS ON PROBLEM 3
- PRESENTATION SCHEDULE (10-12 MINUTE PRESENTATION, 3-5 MINUTES FOR QUESTIONS)
- FINAL: 3 HOURS, SIMILAR TO MIDTERM (CT, MRI, ULTRASOUND)

ULTRASOUND

- PUBLICATION OF "THE THEORY OF SOUND" BY LORD RAYLEIGH IN 1877
- DISCOVERY OF THE PIEZOELECTRIC EFFECT BY PIERRE CURIE IN 1880
- IN 1942, TWO AUSTRALIAN BROTHERS USED TRANSMISSION OF SOUND THROUGH THE BRAIN TO LOCATE TUMORS

CHARACTERISTIC IMPEDANCE:

$$Z \equiv \frac{p}{v} = \rho c$$

p ← ACOUSTIC PRESSURE; v: SPEED OF DISPLACED PARTICLES; ρ: DENSITY; c: SPEED OF WAVE

AS WITH ANY IMPEDANCE IN PHYSICS, Z IS THE RATIO OF THE DRIVING FORCE (PRESSURE p) TO THE PARTICLE VELOCITY (v).

1D WAVE EQUATION:

$$\frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

p ← ACOUSTIC PRESSURE; c: SPEED OF WAVE

GENERAL SOLUTION:

$$p(z,t) = \underbrace{\phi_f\left(t - \frac{z}{c}\right)}_{\text{FORWARD TRAVELING WAVE}} + \underbrace{\phi_b\left(t + \frac{z}{c}\right)}_{\text{BACKWARD TRAVELING WAVE}}$$

ANY WAVEFORM THAT CAN BE GENERATED WILL PROPAGATE THROUGH MATTER!

SPHERICAL WAVES:

SPHERICAL WAVE EQN:

$$\boxed{\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}}$$

⇐ DISTANCE FROM SOURCE OR DISTURBANCE

GENERAL SOLN:

$$p(r,t) = \frac{1}{r}\underbrace{\phi_o\left(t - \frac{r}{c}\right)}_{\text{OUTWARD TRAVELING WAVE}} + \cancel{\frac{1}{r}\underbrace{\phi_i\left(t + \frac{r}{c}\right)}_{\text{INWARD TRAVELING WAVE}}}$$

TYPICALLY CAN'T GENERATE!

WAVE LOSES AMPLITUDE (INCREASING SURFACE AREA)!

ATTENUATION

- REFERS TO THE LOSS OF ACOUSTIC ENERGY OF THE ULTRASONIC WAVE DURING PROPAGATION.
- IN TISSUES, ATTENUATION IS MAINLY DUE TO THE CONVERSION OF ACOUSTIC ENERGY INTO HEAT BECAUSE OF VISCOSITY.
- IT RESULTS IN AN EXPONENTIAL DECAY OF THE AMPLITUDE OF THE PROPAGATING WAVE. ⇑ PHENOMENOLOGICAL!
- LOSSES COME FROM ABSORPTION, SCATTERING, AND MODE CONVERSION ⇐ LONGITUDINAL WAVES ⇔ SHEAR WAVES

FOR A PLANE WAVE:

$$p(z,t) = \phi_f\left(t - \frac{z}{c}\right)$$

WE ACTUALLY GET:

NO LONGER SATISFIES WAVE EQN!

$$p(z,t) = A_z\,\phi_f\left(t - \frac{z}{c}\right) \Longrightarrow \boxed{A_z = A_0 e^{-\mu_a z}}$$

AMPLITUDE ATTENUATION FACTOR (cm^{-1})

OR "NEPERS PER CM", WHERE A "NEPER" IS A DIMENSIONLESS UNIT DEFINED AS $\ln\left(\frac{A_z}{A_0}\right)$.

WE MORE TYPICALLY USE UNITS OF dB/cm (INSTEAD OF Np/cm). WHEN μ_a IS CONVERTED TO dB/cm, WE CALL IT THE ATTENUATION COEFFICIENT α.

$$\boxed{\alpha = 20(\log_{10} e)\mu_a = 8.686\,\mu_a}$$

SO:

$$\boxed{1\ Np = 8.686\ dB}$$

ATTENUATION IS ROUGHLY LINEARLY PROPORTIONAL TO FREQUENCY:

$$\alpha \approx a f$$

FREQUENCY

TABLE ON PAGE 327 (TABLE 10.2) UNITS ARE: $\frac{dB}{cm \cdot MHz}$

SO PUTTING IT ALL TOGETHER:

$$\boxed{A_z \approx A_0\, e^{-\frac{af}{8.686}z} = A_0\, e^{-\frac{\alpha}{8.686}z} = A_0\, e^{-\mu_a z}}$$

<u>EXAMPLE:</u>

$a = 0.56 \frac{dB}{cm \cdot MHz}$ IN HOMOGENIZED LIVER

HOW FAR WILL A 2 MHz WAVE PROPAGATE BEFORE LOSING HALF ITS AMPLITUDE?

<u>ANSWER:</u>

WE WANT:

$$\frac{A_z}{A_0} = \frac{1}{2} = e^{-\frac{afz}{8.686}}$$

$$\left(\frac{1}{-8.686\ \frac{Np}{\cancel{dB}}}\right)\left(0.56\ \frac{\cancel{dB}}{cm \cdot \cancel{MHz}}\right)\left(2\ \cancel{MHz}\right) z = \ln\left(\frac{1}{2}\right)$$

$$z = \frac{\ln\left(\frac{1}{2}\right)(-8.686)}{(0.56)(2)}\ cm.$$

$$\boxed{z = 5.4\ cm}$$

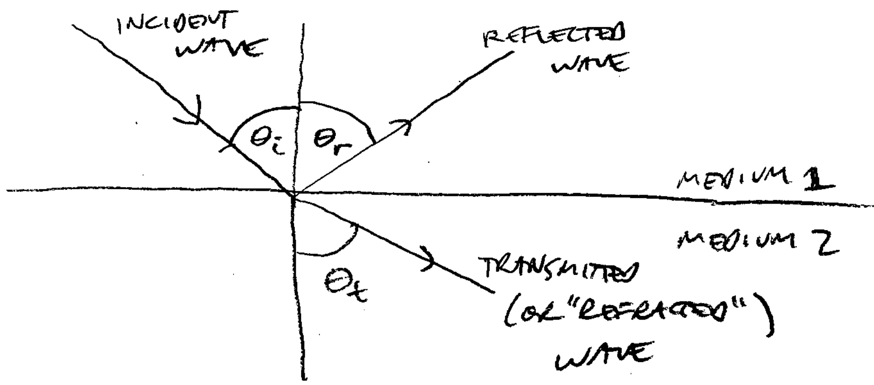
A WORD ON NONLINEARITY BECAUSE I'M A PHYSICIST... :)

- HAD WE GONE THROUGH A DERIVATION OF THE WAVE EQN., WE WOULD HAVE SEEN THAT WE ASSUMED THAT THE ACOUSTIC PRESSURE DISTURBANCES WERE VERY SMALL COMPARED TO THE STATIC ACOUSTIC PRESSURE.
- THIS ALLOWED US TO ONLY KEEP LINEAR TERMS (1ST ORDER TERMS IN A TAYLOR EXPANSION), AND WE GOT TO THE <u>LINEAR WAVE EQUATION</u> (PERFECT ELASTICITY).
- SO-CALLED "FINITE AMPLITUDE" EFFECTS DO OCCUR IN REALITY, LEADING TO DISTORTION OF THE WAVEFORM AS IT PROPAGATES $\Longrightarrow$ GENERATION OF HIGHER HARMONICS.

OUTSIDE OUR SCOPE, BUT JUST WANTED YOU TO BE AWARE.

REFLECTION AND REFRACTION:

CONSIDER A PLANE WAVE INCIDENT ON A BOUNDARY BETWEEN TWO MEDIA AS SHOWN:



OBEY THE LAWS OF GEOMETRIC OPTICS:

$$\boxed{\theta_i = \theta_r}$$

$$\boxed{\frac{\sin \theta_i}{\sin \theta_t} = \frac{c_1}{c_2}}$$

"SNELL'S LAW"

c_1 = SPEED OF SOUND IN MEDIUM 1

c_2 = " IN MEDIUM 2

TABLE 10.1 IS USEFUL FOR SPEEDS, IMPEDENCES, ETC. IN VARIOUS TISSUES

BY REQUIRING CONTINUITY OF ACOUSTIC PRESSURE ACROSS THE INTERFACE AND REQUIRING THAT THE TANGENTIAL PARTICLE MOTION AT THE INTERFACE EQUALS THE SUM OF THE TANGENTIAL PARTICLE MOTION OF THE TRANSMITTED AND REFLECTED WAVES, WE CAN DERIVE (AFTER SOME MATH):

Z'S ARE CHARACTERISTIC IMPEDANCES

$$R \equiv \frac{p_r}{p_i} = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

$$T \equiv \frac{p_t}{p_i} = \frac{2 Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

REFLECTION AND TRANSMISSION COEFFICIENTS

TAKE A LOOK AT EXAMPLE 10.5 IN THE BOOK!

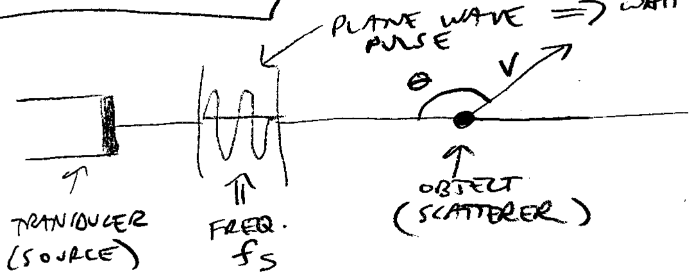
SCATTERING:

- REFLECTIONS DON'T ONLY OCCUR AT TISSUE BOUNDARIES.
- INDIVIDUAL TISSUES ARE INHOMOGENEOUS.
- WE MODEL SMALL "POINT SCATTERERS" THAT ARE EXCITED BY INCIDENT WAVES, AND THEN ACT AS SOURCES OF RETRANSMITTED ULTRASONIC WAVES (RETRANSMITTING THE INCIDENT WAVE EQUALLY IN ALL DIRECTIONS). ⇐ HUYGENS PRINCIPLE
- LARGER SCATTERING STRUCTURES CAN BE MODELLED AS COLLECTIONS OF POINT SCATTERERS.
- FOR SCATTERING BODIES MUCH SMALLER THAN THE WAVELENGTH, SCATTERED WAVELETS MOSTLY INTERFERE CONSTRUCTIVELY.
- MORE COMPLEX FOR LARGE SCATTERING BODIES.

SHOW FIGURES 7.6 AND 7.7 IN SUETENS.

DOPPLER SHIFT:

PLANE WAVE ⇒ WAIT FOR ECHO ("PULSE-ECHO")



MOVING OBJECT OBSERVES FREQUENCY:

$$f_o = \frac{c + v\cos\theta}{c} f_s$$

THIS FREQUENCY IS RETRANSMITTED BY THE MOVING SCATTERER, AND WE OBSERVE A FREQUENCY f_T AT THE RECEIVER (TRANSDUCER) OF:

$$f_T = \left(\frac{c}{c - v\cos\theta}\right) f_o = \left(\frac{\cancel{c}}{c - v\cos\theta}\right)\left(\frac{c + v\cos\theta}{\cancel{c}}\right) f_s$$

$$f_T = \left(\frac{c + v\cos\theta}{c - v\cos\theta}\right) f_s = \left(1 + \frac{2v\cos\theta}{c - v\cos\theta}\right) f_s$$

THE DOPPLER SHIFT IS THEN:

$$f_D \equiv f_T - f_S = \frac{2v\cos\theta}{c - v\cos\theta} f_S$$

IN GENERAL, $v \ll c$, SO:

$$\boxed{f_D \approx \frac{2v\cos\theta}{c} f_S}$$ DOPPLER SHIFT

BEAM PATTERN FORMATION AND FOCUSING

REVIEW SECTION 10.5 IN THE BOOK! I'M NOT GOING INTO MUCH DETAIL HERE.

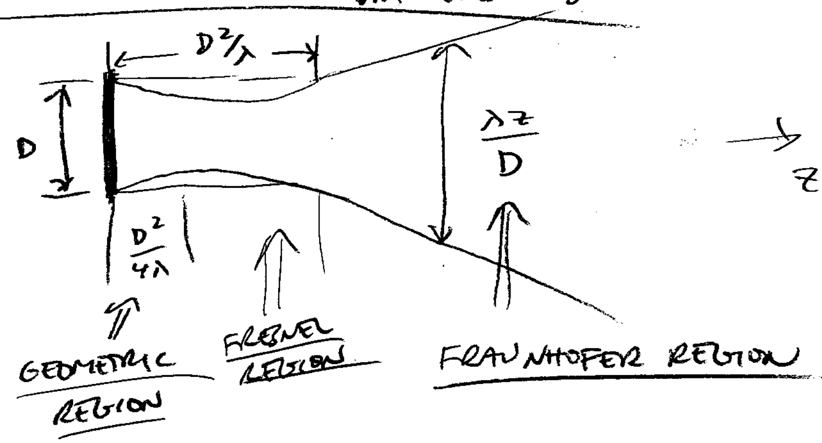
SOME IMPORTANT POINTS:

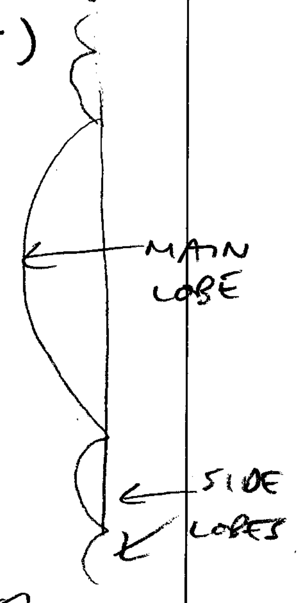
- WE HAVE CONSIDERED PLANE WAVES SO FAR $\Rightarrow$ NOT ENTIRELY TRUE
- "FIELD PATTERN": SPATIAL DISTRIBUTION OF THE ACOUSTIC INTENSITY OF A SINUSOIDALLY EXCITED TRANSDUCER IN STEADY STATE.

$$I = \underbrace{p}_{\text{ACOUSTIC PRESSURE}} \overbrace{v}^{\text{PARTICLE VELOCITY}} = \frac{p^2}{Z}$$ $\Leftarrow$ GOES AS THE SQUARE OF ACOUSTIC PRESSURE

(ANALOGOUS TO POWER IN CIRCUITS)

- FOR A TRANSDUCER OF DIAMETER D:





WE IGNORE SIDE LOBES, BUT THEY CAUSE ARTIFACT SOMETIMES.

GENERATION AND DETECTION OF ULTRASOUND

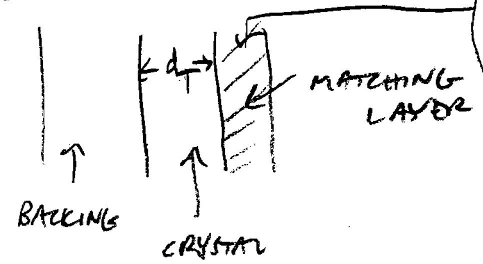
- ULTRASONIC WAVES ARE BOTH GENERATED AND DETECTED BY A PIEZOELECTRIC CRYSTAL, WHICH DEFORMS UNDER THE INFLUENCE OF AN ELECTRIC FIELD (AND VICE VERSA ⇒ IT INDUCES AN ELECTRIC FIELD OVER THE CRYSTAL AFTER DEFORMATION!).

- CRYSTAL IS USED TO MAKE A TRANSDUCER THAT ACTS AS BOTH A TRANSMITTER AND A DETECTOR.

- TWO PIEZOELECTRIC POLYMERS ARE OFTEN USED:
  - PZT (LEAD ZIRCONATE TITANATE)
  - PVDF (POLYVINYLIDENE FLUORIDE)
- CAN BE MANUFACTURED IN NEARLY ANY SHAPE, WITH AXIS OF POLARIZATION IN ALMOST ANY DIRECTION.

FUNDAMENTAL RESONANCE FREQUENCY OF TRANSDUCER:

ULTRASONIC TRANSDUCER



QUARTER WAVELENGTH w/

$$Z = \sqrt{Z_c Z_t}$$

CRYSTAL (Z_c) TISSUE (Z_t)

- PART OF THE ENERGY PRODUCED BY A TRANSDUCER IS REFLECTED BACK TOWARDS THE BACKING.
- THIS REFLECTED WAVE INDUCES ELECTRICAL FIELDS THAT INTERFERE W/ THE DRIVING ELECTRICAL FORCE.
- AMPLITUDE OF VIBRATION IS MAXIMAL WHEN:

$$\boxed{d_T = \frac{1}{2}\lambda_T} \qquad \lambda_T = \frac{c_T}{f_T}$$

RESONANCE CONDITION FOR TRANSDUCER