- FINAL PROJECT PRESENTATIONS: ⇐ BE PUNCTUAL!!!
  - START THURSDAY
  - 10-12 MINUTES + 3-5 MINUTES FOR QUESTIONS! ⇒ I WILL BE RUNNING THIS LIKE A SCIENTIFIC SESSION AT A CONFERENCE
  - TEST YOUR POWERPOINT BEFOREHAND! ARRIVE 10 MINUTES EARLY THE DAY OF YOUR PRESENTATION
  - WE WILL HAVE TO CUT YOU OFF IF YOU GO OVER, AND THAT WILL ADVERSELY AFFECT YOUR GRADE!
- FINAL REPORTS
  - DUE FOR EVERYONE AT MIDNIGHT ON DECEMBER 10^{TH}
  - 2-3 PAGES SUGGESTED, BUT YOU ARE WELCOME TO GO LONGER.
  - PROVIDE REFERENCES!
  - IF PROJECT SCOPE IS LARGE, EITHER
    1. PROVIDE A HIGH-LEVEL SUMMARY + RESULTS, OR
    2. DELVE INTO A NARROWER TOPIC IN MORE DETAIL.

THE PULSE-ECHO EQN. IN ULTRASOUND:

- AN IDEAL ULTRASOUND SYSTEM WOULD RECONSTRUCT & DISPLAY $R(x,y,z)$, THE SPATIAL DISTRIBUTION OF REFLECTIVITY

  ↑ BASED ON SCATTERING, NOT SPECULAR REFLECTION

- TRANSDUCER'S IMPULSE RESPONSE FUNCTION WILL BLUR OUT TRUE REFLECTIVITY

- USE OF ENVELOPE DETECTION WILL CREATE "SPECKLE" (AND BLOTCHINESS IN IMAGES)

- SOME OBJECTS CAUSE SPECULAR REFLECTION, RATHER THAN SCATTER. WHY MIGHT THIS BE A PROBLEM?

DERIVATION OF THE FIELD PATTERN:

FIELD PATTERN IS THE SPATIAL DISTRIBUTION OF THE ACOUSTIC INTENSITY OF TRANSDUCER UNDERGOING STEADY-STATE SINUSOIDAL EXCITATION.

WHAT IS THE FIELD PATTERN OF A FLAT, VIBRATING PLATE?

IN THE FAR FIELD, OR FRAUNHOFER ZONE, WE USE THE DIFFRACTION FORMULATION

- WE IGNORE THE ELECTRONICS, THE ELECTRICAL - MECHANICAL INTERFACE TO THE TRANSDUCER, AND THE MECHANICAL - ACOUSTICAL INTERFACE TO THE MEDIUM.

NARROWBAND PULSE:

- WE MODEL THE PRESSURE SIGNAL OF A TRANSMITTED PULSE AS:

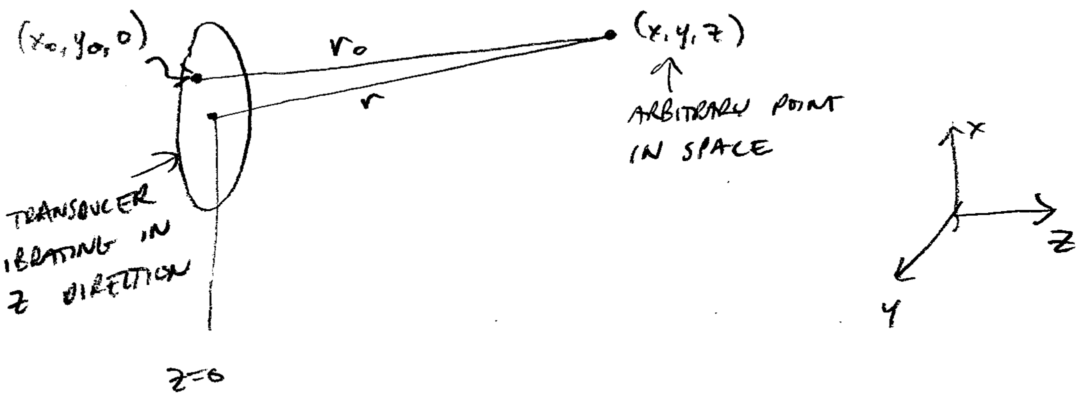
$$\underbrace{n(t)}_{\text{COMPLEX SIGNAL}} = \tilde{n}(t) e^{-j2\pi f_0 t}$$

$$\underbrace{\tilde{n}(t)}_{\text{COMPLEX ENVELOPE}} = \underbrace{n_e(t)}_{\text{ENVELOPE}} e^{j\phi}$$

- ENVELOPE IS ASSUMED TO BE LONG COMPARED TO PERIOD $\frac{1}{f_0}$.
  - THIS IS STILL GENERALLY A GOOD APPROXIMATION IN MEDICAL ULTRASOUND SYSTEMS, EVEN THOUGH WE TYPICALLY HAVE ONLY 3-5 OSCILLATIONS PER ENVELOPE.

$\boxed{\text{ENVELOPE IS } |n(t)|}$

- THE ACTUAL SIGNAL IS: $Re\{n(t)\}$, BUT WE KEEP THE COMPLEX MODEL ⇒ MAKES IT EASIER TO DESCRIBE THE ENVELOPE, WHICH IS THE OUTPUT OF AN ULTRASOUND SYSTEM.



⇒ $r_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}$

⇒ EACH POINT ON TRANSDUCER $(x_0, y_0, 0)$ PRODUCES AN INDEPENDENT ACOUSTIC WAVE!

⇒ THE LINEAR SUPERPOSITION OF ALL THESE WAVES COMPRISES THE FIELD PATTERN OF THE FLAT PLATE!

EACH POINT PRODUCES AN ACOUSTIC WAVE:

$$p(x,y,z,t) = \frac{z}{r_0^2} n\left(t - \frac{r_0}{c}\right)$$

ON THE Z AXIS, THIS LOOKS LIKE A SPHERICAL WAVE, BUT GOES TO ZERO AT A 90° ANGLE TO THE SOURCE.

TOTAL PRESSURE AT (x,y,z) IS GIVEN BY INTEGRATING OVER THE SOURCE:

DIPOLE PATTERN

$$p(x,y,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x_0,y_0) \frac{z}{r_0^2} n\left(t - \frac{r_0}{c}\right) dx_0\, dy_0$$

TRANSDUCER "INDICATOR FUNCTION" (1 WHERE TRANSDUCER IS, 0 ELSEWHERE)

IF WE HAVE A SPHERICAL WAVE SCATTERER OF STRENGTH $R(x,y,z)$ AT (x,y,z), THEN THE PRESSURE AT THE TRANSDUCER POINT (x_0', y_0') FROM THE SCATTERED WAVE WILL BE:

$$P_s(x_0', y_0', t) = R(x,y,z) \frac{1}{r_0'} p\left(x,y,z,t - \frac{r_0'}{c}\right)$$

STRENGTH OF SCATTERER

PURE SPHERICAL WAVE

WHERE r_0' IS THE DISTANCE FROM SCATTERER AT (x,y,z) TO $(x_0', y_0', 0)$.

- TRANSDUCER IS SENSITIVE ONLY IN Z DIRECTION, SO WE WEIGHT BY A COSINE (DIPOLE PATTERN).

- INTEGRATING OVER TRANSDUCER FACE:

$$r(x,y,z,t) = K \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x_0', y_0') \frac{z}{r_0'} P_s(x_0', y_0', t)\, dx_0'\, dy_0'$$

RECEIVED ELECTRICAL WAVEFORM

GAIN FACTOR FOR HARDWARE CONSIDERATIONS

COSINE (DIPOLE) SENSITIVITY

PUTTING IT TOGETHER:

$$r(x,y,z,t) = K\,R(x,y,z) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x_0',y_0')\frac{z}{r_0'^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x_0,y_0)\frac{z}{r_0^2}\, n\left(t-\frac{r_0}{c}-\frac{r_0'}{c}\right) dx_0\,dy_0\,dx_0'\,dy_0'$$

YUCK! FORTUNATELY, WE NOW START MAKING ASSUMPTIONS.

PLANE WAVE APPROXIMATION:

EXCITATION PULSE ARRIVES AT ALL POINTS IN A GIVEN Z-PLANE SIMULTANEOUSLY.

$$n\left(t-\frac{r_0}{c}-\frac{r_0'}{c}\right) \approx n\left(t-\frac{2z}{c}\right)e^{-j2\pi f_0\left(t-\frac{r_0}{c}-\frac{r_0'}{c}\right)}$$

QUADRUPLE INTEGRAL SPLITS INTO TWO IDENTICAL DOUBLE INTEGRALS:

COMPLEX RECEIVED SIGNAL FOR A SINGLE SCATTERER

$$r(x,y,z,t) = K\,R(x,y,z)\,n\left(t-\frac{2z}{c}\right)\left[g(x,y,z)\right]^2$$

WHERE:

USING WAVE NUMBER INSTEAD OF f_0.

FIELD PATTERN

$$\Longrightarrow g(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(x_0,y_0)\,\frac{z}{r_0^2}\,e^{jk(r_0-z)}\,dx_0\,dy_0$$

BASIC PULSE-ECHO SIGNAL EQN.

$$r(t) = \int_{z=0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(x,y,z,t)\,dx\,dy\,dz$$

$$r(t) = K\iiint R(x,y,z)\,n\left(t-\frac{2z}{c}\right)e^{-2\mu_a z}\left[g(x,y,z)\right]^2 dx\,dy\,dz$$

DISTRIBUTION OF SCATTERERS

ROUND TRIP ATTENUATION FACTOR.

TO MAKE TRIPLE INTEGRAL MORE TRACTABLE:

PARAXIAL APPROXIMATION:

$r_0 \approx z$ ⟸ PRIMARILY INTERESTED IN FIELD PATTERN NEAR z AXIS. (TRANSDUCER AXIS)

CAN ONLY BE APPLIED TO AMPLITUDE TERMS!
(PHASE IS STILL IMPORTANT.)

$$g(x,y,z) \approx \frac{1}{z} \iint s(x_0,y_0)\, e^{jk(r_0 - z)}\, dx_0\, dy_0$$

FRESNEL APPROXIMATION:

SIMPLIFIES PHASE TERMS:

$$r_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2} = z\sqrt{1 + \underbrace{\frac{(x-x_0)^2}{z^2} + \frac{(y-y_0)^2}{z^2}}}$$

FOR BIG z:

MUCH LESS THAN 1!

$$r_0 \approx z\left(1 + \frac{1}{2}\left[\frac{(x-x_0)^2}{z^2} + \frac{(y-y_0)^2}{z^2}\right]\right)$$

$$\approx z + \frac{(x-x_0)^2}{2z} + \frac{(y-y_0)^2}{2z}$$

APPLY THIS TO PHASE TERM IN $g(x,y,z)$:

$$g(x,y,z) \approx \frac{1}{z} \iint s(x_0,y_0)\, e^{jk\left(\frac{(x-x_0)^2}{2z} + \frac{(y-y_0)^2}{2z}\right)}\, dx_0\, dy_0$$

CONVOLUTION INTEGRAL!

$$\boxed{g(x,y,z) = \frac{1}{z}\, s(x,y) \underset{2D}{\underset{\pi}{*}} e^{jk(x^2+y^2)/2z}}$$

FRAUNHOFER APPROXIMATION:

EXPANDING:

$$r_0 \approx z + \frac{(x-x_0)^2}{2z} + \frac{(y-y_0)^2}{2z}$$

$$= z - \frac{xx_0}{z} - \frac{yy_0}{z} + \frac{x^2+y^2}{2z} + \frac{x_0^2+y_0^2}{2z}$$

CAN FURTHER APPROXIMATE $g(x,y,z)$

FOR z LARGE:

$$e^{jk(x_0^2+y_0^2)/2z} \approx 1$$

$$g(x,y,z) \approx \frac{1}{z} e^{jk(x^2+y^2)/2z} S\left(\frac{x}{\lambda z}, \frac{y}{\lambda z}\right)$$

FOURIER TRANSFORM OF TRANSDUCER!