ECEn 682R	LECTURE 3	9/8/2009

- HOMEWORK #1 WILL BE POSTED BY END OF DAY TODAY
- WEBSITE & SYLLABUS UPDATED SHORTLY

SYSTEMS:

- TRANSFORMATION OF AN INPUT SIGNAL TO AN OUTPUT SIGNAL.
- DIFFERENT KINDS:
  - CONTINUOUS TO CONTINUOUS
  - DISCRETE TO CONTINUOUS
  - CONTINUOUS TO DISCRETE
  - DISCRETE TO DISCRETE

NOTATION:

$$g(x,y) = \mathcal{S}[f(x,y)]$$ ⇐ BOOK ⟵ TOO GENERAL TO BE USEFUL IN PRACTICE!

OR

$$f(x,y) \Longrightarrow g(x,y)$$

WE SOMETIMES USE:

$\mathcal{S}[f(x,y)]$ TO DENOTE THE OUTPUT AT POINT (x,y).
WHY IS THIS A NOTATIONAL STRETCH?

LINEARITY:

A SYSTEM IS LINEAR IF:

$$\mathcal{S}\left[w_1 f_1(x,y) + w_2 f_2(x,y)\right] = w_1 \mathcal{S}\left[f_1(x,y)\right] + w_2 \mathcal{S}\left[f_2(x,y)\right]$$

FOR ANY $f_1(x,y)$ AND $f_2(x,y)$. MORE GENERALLY:

$$\mathcal{S}\left[\sum_{k=1}^{K} w_k f_k(x,y)\right] = \sum_{k=1}^{K} w_k \mathcal{S}\left[f_k(x,y)\right]$$

IN OTHER WORDS, SUPERPOSITION HOLDS.

NOTE THAT LINEARITY IN THE SYSTEMS CASE IMPLIES THAT A ZERO INPUT YIELDS A ZERO OUTPUT! (UNLIKE A LINEAR EQN.)

- LINEARITY MAKES OUR LIFE EASIER, AS WE'LL SEE.
- MANY SYSTEMS IN NATURE (AND MEDICAL IMAGING) CAN BE REASONABLY APPROXIMATED BY LINEAR SYSTEMS

THE POINT SPREAD FUNCTION:

I MENTIONED EARLIER THAT THE OUTPUT OF A SYSTEM WHEN THE INPUT IS A DELTA FUNCTION IS IMPORTANT. WE DENOTE:

$$h(x,y;x_0,y_0) = \mathcal{S}\left[\delta_{x_0 y_0}(x,y)\right]$$

$\boxed{\text{PSF IS 4D! :(}}$

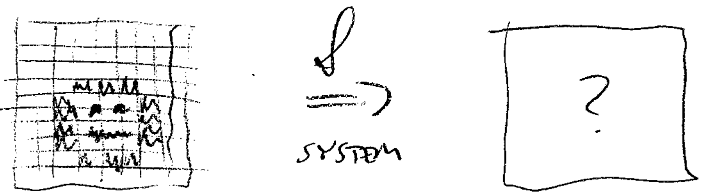
WHAT DOES THIS MEAN? WELL, $h(x,y;x_0,y_0)$ IS THE RESPONSE, OR OUTPUT OF A SYSTEM TO THE SHIFTED DELTA INPUT $\delta_{x_0 y_0}(x,y)$!

IMPULSE RESPONSE OR POINT SPREAD FUNCTION!

$\Longrightarrow$ LET'S GET SOME INTUITION IN THE DISCRETE CASE FIRST.

IMAGE

OUTPUT?



WHAT IF I KNOW:

1. THE SYSTEM IS LINEAR
2. THE PSF

etc.

⋮

NOW EXTEND TO CONTINUOUS:

REMEMBER:

$$f(x_0,y_0) = \iint f(x,y)\,\delta(x-x_0,\, y-y_0)\,dx\,dy$$

$$g(x,y) = \mathcal{S}\left[f(x,y)\right] = \mathcal{S}\left[\iint f(x_0,y_0)\,\delta(x-x_0,y-y_0)\,dx_0\,dy_0\right]$$

$$= \iint \mathcal{S}\left[f(x_0,y_0)\,\delta(x-x_0,y-y_0)\,dx_0\,dy_0\right]$$

$$= \iint f(x_0,y_0)\,\mathcal{S}\left[\delta(x-x_0,y-y_0)\right]dx_0\,dy_0 = \iint f(x_0,y_0)\,h(x,y;x_0,y_0)\,dx_0\,dy_0$$

$\boxed{\text{SUPERPOSITION INTEGRAL}}$

ONLY NEED TO KNOW THE PSF OF LINEAR SYSTEM TO KNOW OUTPUT TO ANY INPUT!

SHIFT INVARIANCE:

KNOWING THE PSF FOR ALL SHIFTS IS A PAIN.

WHAT IF:

GIVEN: $g(x,y) = \mathcal{S}[f(x,y)]$

THEN:

$$\boxed{g(x-x_0, y-y_0) = \mathcal{S}[f(x-x_0, y-y_0)]}$$ ⟸ SHIFT INVARIANCE!

HOW DOES THIS SIMPLIFY THE PSF?

4D ⟹ 2D.

NOW ALL WE NEED TO KNOW IS RESPONSE OF THE SYSTEM TO:

$$\delta(x,y)$$

$$h(x,y) = \mathcal{S}[\delta(x,y)]$$

$$\mathcal{S}[\delta(x-x_0, y-y_0)] = h(x-x_0, y-y_0) \checkmark$$

SO WE CAN MEASURE THE PSF AT ANY POINT IN THE FIELD OF VIEW (FOV) OF AN IMAGING SYSTEM.

CONVOLUTION:

LSI SYSTEM: LINEAR & SHIFT INVARIANT SYSTEM

LET'S GO BACK TO OUR SUPERPOSITION INTEGRAL:

NOW WE HAVE:

$$g(x,y) = \iint f(x_0, y_0)\, h(x-x_0, y-y_0)\, dx_0\, dy_0$$

OR:

$$\boxed{g(x,y) = h(x,y) * f(x,y)}$$ OUR OLD FRIEND CONVOLUTION!

- CASCADED SYSTEMS
- PARALLEL SYSTEMS

- SEPARABLE SYSTEMS:

$$h(x,y) = h_1(x)h_2(y)$$ P- 31 PICTURE

- STABILITY:

BIBO

$|f(x,y)| \leq B < \infty$ FOR ALL x, y

IMPLIES

$|g(x,y)| \leq B' < \infty$ FOR ALL x, y