- HW 1 POSTED
- WEBSITE UPDATED
- HW HELP SESSIONS

THE 2D FOURIER TRANSFORM

TRANSFORM: ALTERNATE REPRESENTATION OF SAME INFO.

- RECALL THE CONVOLUTION EQUATION:

$$g(x,y) = \iint f(x_0, y_0)\, h(x-x_0, y-y_0)\, dx_0\, dy_0$$

THIS WAS OBTAINED BY DECOMPOSING OUR SIGNAL $f(x,y)$ INTO WHAT BASIS SET? $\rightarrow$ 2D IMPULSES

- WE CAN ALSO DECOMPOSE A SIGNAL IN TERMS OF COMPLEX EXPONENTIAL SIGNALS:

$$e^{j2\pi(\underbrace{k_x}_{u} x + \underbrace{k_y}_{v} y)}$$

IT CAN BE SHOWN THAT IF:

$$\boxed{F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy}$$ $\Leftarrow$ 2D FOURIER TRANSFORM

THEN:

$$\boxed{f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv}$$ $\Leftarrow$ INVERSE 2D FOURIER TRANSFORM

INTUITION: $f(x,y)$ IS AN IMAGE.
$F(u,v)$ IS AN IMAGE, AND IS AN ALTERNATE REPRESENTATION OF $f(x,y)$
- TALK IN 2D DISCRETE TO GIVE INTUITION.

DOES THE FOURIER TRANSFORM INTEGRAL ALWAYS EXIST?

IF $f(x,y)$ IS ABSOLUTELY INTEGRABLE AND CONTINUOUS, FT EXISTS.

WHY IS IT USEFUL FOR LSI SYSTEMS?

IN PRACTICE, THE FT ALLOWS US TO SEPARATELY CONSIDER THE ACTION OF AN LSI SYSTEM ON EACH SPATIAL FREQUENCY.

(FOURIER TRANSFORM IS COMPLEX VALUED ⟹ MAGNITUDE & PHASE)

$$\underbrace{\underset{\substack{\Uparrow\\ \text{MAGNITUDE SPECTRUM}}}{|F(u,v)|} \quad \underset{\substack{\Uparrow\\ \text{PHASE SPECTRUM}}}{\angle F(u,v)}}_{\text{BOTH REQUIRED TO UNIQUELY DETERMINE } f(x,y)} \qquad \underbrace{|F(u,v)|^2}_{\text{POWER SPECTRUM}}$$

BASIC FOURIER TRANSFORM PAIRS: $\boxed{\text{p. 34, TABLE 2.1}}$

GO THROUGH TABLE.

AS AN EXAMPLE, COMPUTE FT OF:

$\delta(x,y)$

$e^{j2\pi(u_0 x + v_0 y)}$

$1 \Leftarrow$ CONSTANT DOESN'T VARY IN SPACE. ALL DC COMPONENT.

$$1 \Longleftrightarrow \delta(u,v)$$

$\boxed{\text{SHOW COMPUTER IMAGES!}}$ ⟸ SENT AN EMAIL W/ URL REFERENCE

PROPERTIES OF THE FOURIER TRANSFORM: (TABLE ON P. 38)

LINEARITY:

I HAVE TWO SIGNALS, $f(x,y)$ AND $g(x,y)$. I KNOW THE FT OF EACH:

$$f(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} F(u,v)$$
$$g(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} G(u,v)$$

WHAT IS THE FT OF $a_1 f(x,y) + b_1 g(x,y)$?

$$\boxed{a_1 f(x,y) + b_1 g(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} a_1 F(u,v) + a_2 G(u,v)}$$

TRANSLATION:

IF:

$$f(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} F(u,v)$$

THEN:

$$\boxed{f(x-x_0, y-y_0) \overset{\mathcal{F}}{\Longleftrightarrow} F(u,v)\, e^{-j2\pi(ux_0 + vy_0)}}$$

WHAT IS THIS?
LINEAR PHASE TERM IN
u DIRECTION &
v DIRECTION!

CONJUGATION AND CONJUGATE SYMMETRY:

$$f(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} F(u,v)$$

$$\boxed{f^*(x,y) \overset{\mathcal{F}}{\Longleftrightarrow} F^*(-u,-v)}$$ ⟸ CONJUGATION

WHAT IF $f(x,y)$ IS REAL VALUED?

IF: $f^*(x,y) = f(x,y)$

$$\boxed{F(u,v) = F^*(-u,-v)}$$ CONJUGATE SYMMETRY!