- HW 1 DUE TODAY AT MIDNIGHT
- HW 2 WILL BE POSTED BY END TODAY
- QUESTIONS?

THE 2D FOURIER TRANSFORM: PROPERTIES (CONT.)

f(x,y) REAL VALUED:

$$Re\{F(u,v)\} = Re\{F(-u,-v)\}$$ REAL PART SYMMETRIC

$$Im\{F(u,v)\} = -Im\{F(-u,-v)\}$$ IMAGINARY PART ANTISYMMETRIC

$$|F(u,v)| = |F(-u,-v)|$$ MAGNITUDE SYMMETRIC

$$\angle F(u,v) = -\angle F(-u,-v)$$ PHASE ANTISYMMETRIC

SCALING:

IF:

$$f(x,y) \overset{\mathcal{F}}{\longleftrightarrow} F(u,v)$$

THEN:

$$f(ax,by) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

SHRINKING IN SPACE IS MAGNIFICATION IN SPATIAL FREQUENCY.

(SMALLER DETAIL ⇒ HIGHER SPATIAL FREQUENCIES)

IF $a = b = -1$:

$$f(-x,-y) \overset{\mathcal{F}}{\longleftrightarrow} F(-u,-v)$$

ROTATION:

WE SAW THIS LAST TIME. ROTATION OF f(x,y) ROTATES F(u,v) BY SAME AMOUNT.

CONVOLUTION:

LET:

$$f(x,y) \overset{\mathcal{F}}{\longleftrightarrow} F(u,v)$$
$$g(x,y) \overset{\mathcal{F}}{\longleftrightarrow} G(u,v)$$

THEN:

$$\boxed{f(x,y) * g(x,y) \overset{\mathcal{F}}{\longleftrightarrow} F(u,v)G(u,v)}$$

"CONVOLUTION THEOREM"

EXAMPLE:

$$f(x,y) = \mathrm{sinc}(Ux, Vy) \qquad 0 < V \leq U$$

$$g(x,y) = \mathrm{sinc}(Vx, Uy)$$

WHAT IS:

$$f(x,y) * g(x,y) \ ?$$

SOLN:

$$F(u,v) = \frac{1}{UV}\,\mathrm{rect}\left(\frac{u}{U}, \frac{v}{V}\right)$$

$$G(u,v) = \frac{1}{UV}\,\mathrm{rect}\left(\frac{u}{V}, \frac{v}{U}\right)$$

$$F(u,v)\,G(u,v) = \frac{1}{U^2V^2}\,\mathrm{rect}\left(\frac{u}{V}, \frac{v}{V}\right)$$

$$\Big\Updownarrow \mathcal{F}^{-1}$$

$$f(x,y) * g(x,y) = \frac{1}{U^2}\,\mathrm{sinc}(Vx, Vy)$$

PRODUCT:

$$f(x,y)\,g(x,y) \xleftrightarrow{\mathcal{F}} F(u,v) * G(u,v)$$

SEPARABLE SIGNAL:

IF: $f(x,y) = f_1(x)\,f_2(y)$ (i.e., $f(x,y)$ IS SEPARABLE)

THEN:

$$f(x,y) \xleftrightarrow{\mathcal{F}} \underbrace{F_1(u)}_{\mathcal{F}\{f_1(x)\}}\ \underbrace{F_2(u)}_{\mathcal{F}\{f_2(y)\}}$$

PARSEVAL'S THEOREM

$$\boxed{\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |f(x,y)|^2\,dx\,dy}_{\text{TOTAL ENERGY IN } f(x,y)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |F(u,v)|^2\,du\,dv}$$

COMPUTING A 2D FT:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,e^{-j2\pi(ux+vy)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty} \underbrace{\left[\int_{-\infty}^{\infty} f(x,y)\,e^{-j2\pi ux}\,dx\right]}_{\substack{\text{DO THIS FIRST,} \\ \text{AND YOU END UP} \\ \text{WITH A FUNCTION} \\ \text{OF } y!}} e^{-j2\pi vy}\,dy$$

TRANSFER FUNCTION:

CONSIDER AN LSI SYSTEM. RECALL THAT IF WE KNOW THE OUTPUT OF AN LSI SYSTEM WHEN THE INPUT IS $\delta(x,y)$, WE KNOW EVERY THERE IS TO KNOW ABOUT THE SYSTEM. WE CALL THIS THE PSF:

$$\delta(x,y) \xRightarrow{\mathcal{L}} h(x,y)$$

WE SHOWED THAT WE CAN NOW FIND THE OUTPUT OF THE SYSTEM FOR 'ANY' INPUT $f(x,y)$ USING:

$$g(x,y) = f(x,y) * h(x,y)$$

WHERE: $f(x,y) \underset{h(x,y)}{\xRightarrow{\mathcal{L}}} g(x,y)$

NOW, BY THE CONVOLUTION THEOREM, WE CAN WRITE:

$$\boxed{G(u,v) = F(u,v)\underbrace{H(u,v)}}$$

"TRANSFER FUNCTION"
"FREQUENCY RESPONSE"
"OPTICAL TRANSFER FUNCTION (OTF)"

2D FT OF PSF!

WHAT IS THE TRANSFER FUNCTION TELLING US?

LET THE INPUT BE A COMPLEX EXPONENTIAL INPUT:

$$e^{j2\pi(u_0 x + v_0 y)} \underset{h(x,y)}{\xRightarrow{'\mathcal{L}'}} \underset{g(x,y)}{?}$$

SPATIAL FREQUENCIES

IF I PUT THIS INTO AN LSI SYSTEM, WHAT DO I GET OUT?

$$g(x,y) = e^{j2\pi(u_0 x + v_0 y)} * h(x,y)$$

$$G(u,v) = \delta(u-u_0, v-v_0)\, H(u,v)$$

$$G(u,v) = \underbrace{H(u_0,v_0)}_{\text{CONSTANT}}\, \delta(u-u_0, v-v_0)$$

AND LOW-PASS FILTER p. 4J

$$g(x,y) = \underbrace{H(u_0,v_0)}_{\substack{\text{CONSTANT} \\ (\text{"EIGENVALUE"})}}\, \underbrace{e^{j2\pi(u_0 x + v_0 y)}}_{\text{THIS WAS OUR ORIGINAL INPUT!}} \Leftarrow \text{"EIGENFUNCTION"}$$

INTUITION OF EQUALIZER