- CHANGE IN HW DUE DATE (#2 AVAILABLE LATER TODAY)
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SAMPLING:

- "DISCRETIZATION"
- TRANSFORM CONTINUOUS DATA INTO A SERIES OF NUMBERS
- COMMONLY DONE BY SAMPLING ON A CARTESIAN GRID ("RECTANGULAR SAMPLING") IN 2D.

SAMPLING PERIOD:

Δx APART IN x
Δy APART IN y

SAMPLING FREQUENCY?

$\frac{1}{\Delta x}$ IN x

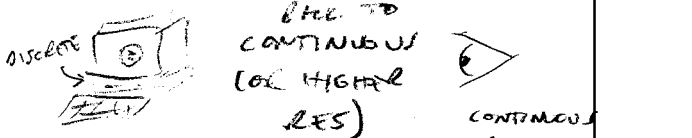
$\frac{1}{\Delta y}$ IN y

NOTATION:

$$f_d(m,n) = f(m\Delta x, n\Delta y)$$

$f_d(m,n)$: DISCRETE VERSION OF $f(x,y)$, SAMPLED EVERY Δx IN x AND Δy IN y

$f(m\Delta x, n\Delta y)$: PICKING OFF VALUES OF $f(x,y)$



UNFORTUNATELY, THIS ISN'T VERY MATHEMATICALLY USEFUL. :^ (AS WE'LL SEE IN A MOMENT.)

WHEN WE TRY TO RECONSTRUCT, THAT IS.

THE GOAL: KEEP AS FEW SAMPLES AS POSSIBLE TO ACCURATELY REPRESENT THE SIGNAL (IMAGE).

QUESTION: CAN WE EVER EXACTLY RECOVER $f(x,y)$ (THE CONTINUOUS SIGNAL) FROM $f_d(m,n)$ (THE SAMPLED VERSION)?

WE'LL SEE... :)

I MENTIONED THAT THE MATHEMATICAL DESCRIPTION

$$f_d(m,n) = f(m\Delta x, n\Delta y)$$

DOESN'T GIVE US MUCH INSIGHT INTO WHAT HAPPENS WHEN WE SAMPLE AND THEN TRY TO RECONSTRUCT, WHICH WE TYPICALLY NEED TO DO FOR REAL SYSTEMS.

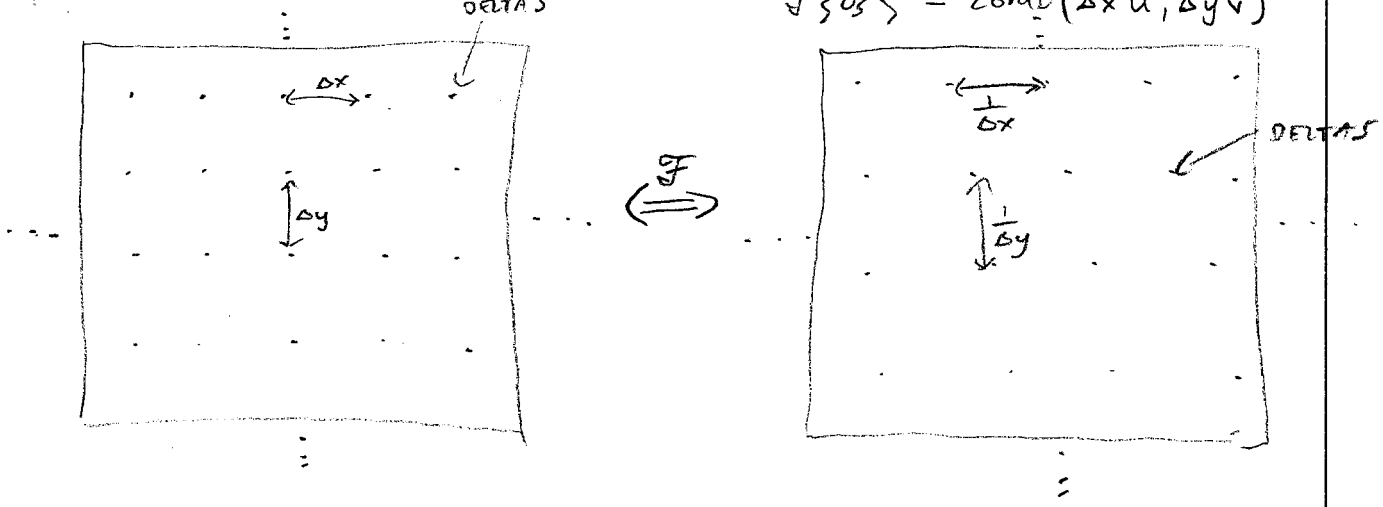
SAMPLING SIGNAL MODEL:

RECALL OUR "SAMPLING FUNCTION":

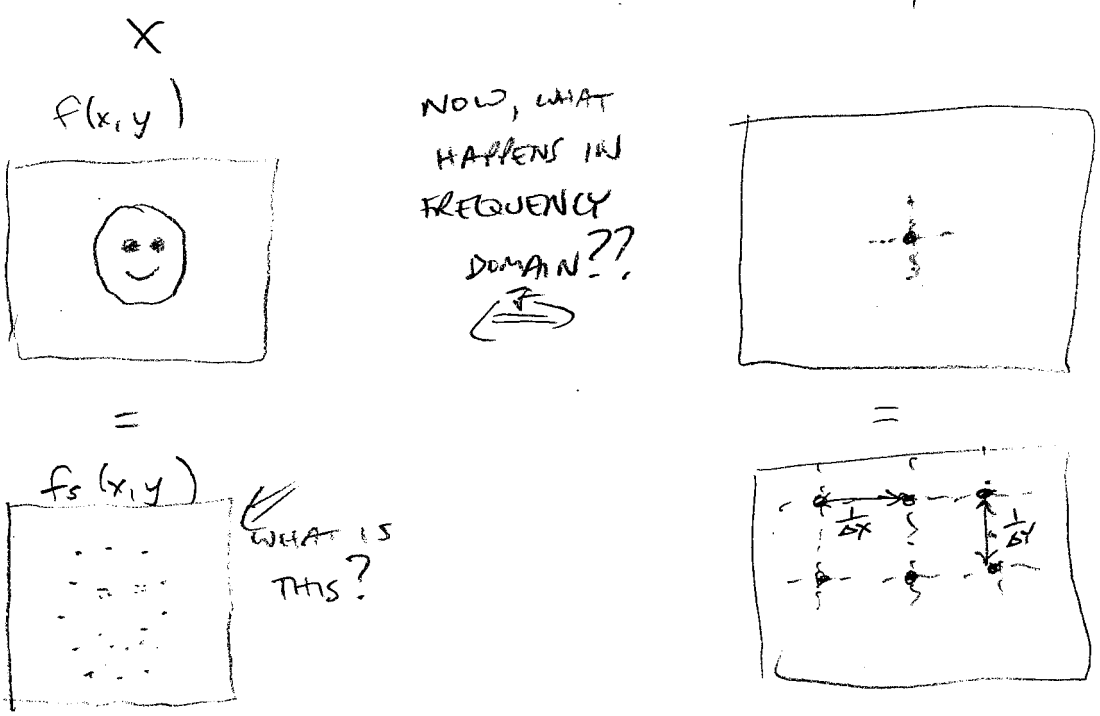
$$\delta_s(x,y;\Delta x,\Delta y) = \frac{1}{\Delta x \Delta y}\,\mathrm{comb}\left(\frac{x}{\Delta x},\frac{y}{\Delta y}\right)$$

JUST FOR KICKS:

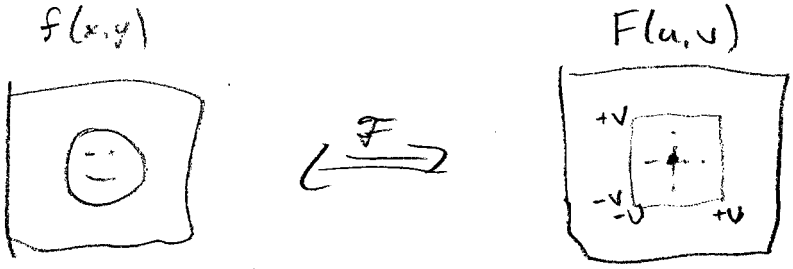
$$\mathcal{F}\{\delta_s\} = \mathrm{comb}(\Delta x\, u, \Delta y\, v)$$



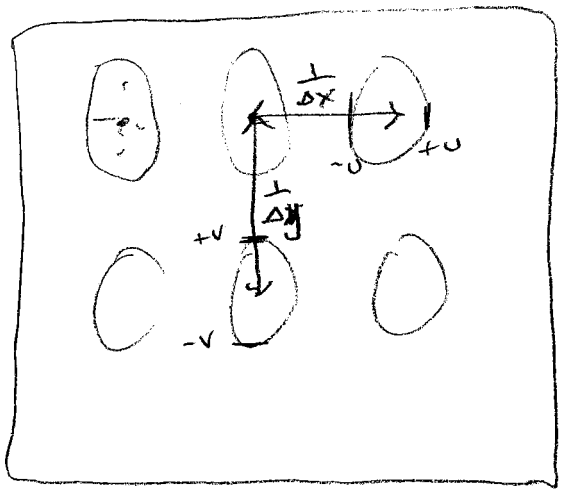
WE MODEL SAMPLING AS A MULTIPLICATION OF AN IMAGE $f(x,y)$ BY THIS SAMPLING FUNCTION: ✳



NOW, WHAT IF OUR $f(x,y)$ IS "BAND LIMITED"?



WHAT IS CONDITION FOR THE SPECTRA TO NOT OVERLAP?



$$\frac{1}{\Delta x} > 2U$$

$$\frac{1}{\Delta y} > 2V$$

"NYQUIST SAMPLING RATE"

OR

$$\Delta x < \frac{1}{2U}$$

$$\Delta y < \frac{1}{2V}$$

"NYQUIST SAMPLING PERIOD"!

RECONSTRUCTION:

ANTI-ALIASING FILTER: