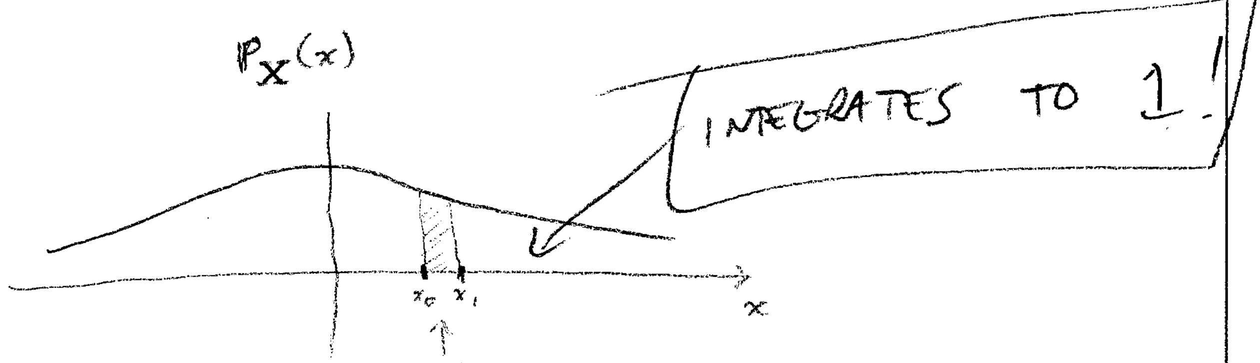
CONTINUOUS RANDOM VARIABLE:

- UNIQUELY SPECIFIED BY ITS "PROBABILITY DENSITY FUNCTION" OR pdf (LOWERCASE):



PROBABILITY THAT A MEASUREMENT OF X WILL BE BETWEEN x_0 AND x_1

CAN CHARACTERIZE RANDOM VARIABLES BY MEAN AND VARIANCE (OR STANDARD DEVIATION). (BUT NOT COMPLETELY!)

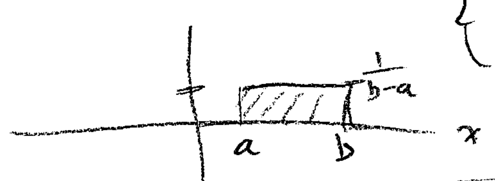
$$\boxed{\mu_X = E(X) = \int_{-\infty}^{\infty} x \, p_X(x)\, dx} \quad \text{MEAN}$$

$$\boxed{\sigma_X^2 = \mathrm{Var}(X) = E\left((X-\mu_X)^2\right) = \int_{-\infty}^{\infty} (x-\mu_X)^2 \, p_X(x)\, dx} \quad \text{VARIANCE}$$

A COUPLE OF IMPORTANT CONTINUOUS RANDOM VARIABLES:

UNIFORM:

$$p_X(x) = \begin{cases} \frac{1}{(b-a)}, & a \le x < b \\ 0 & \text{else} \end{cases}$$



$$\boxed{\begin{aligned} \mu_X ? &= \frac{a+b}{2} \\ \sigma_X^2 &= \frac{(b-a)^2}{12} \end{aligned}}$$

GAUSSIAN RANDOM VARIABLE:

OR NORMAL RANDOM VARIABLE

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$$\boxed{\begin{aligned} \mu_X &= \mu \\ \sigma_X^2 &= \sigma^2 \end{aligned}}$$

IN THE CASE OF A GAUSSIAN, IT IS UNIQUELY SPECIFIED BY MEAN AND VARIANCE!

DISCRETE RANDOM VARIABLE:

WHEN X CAN ONLY ASSUME DISCRETE VALUES, WE DEFINE A DISCRETE PROBABILITY MASS FUNCTION ~~(PMF)~~ (PMF):

$$Pr[X = x_i], \text{ FOR } i = 1, 2, \ldots, k$$

k CAN GO TO ∞!

$$0 \leq Pr[X = x_i] \leq 1$$

$$\sum_{i=1}^{k} Pr[X = x_i] = 1$$

POISSON:

X TAKES VALUES $0, 1, \ldots$ (INTEGERS)

PROBABILITY OF AN INTEGER k IS:

$$\boxed{Pr[X = k] = \frac{a^k}{k!} e^{-a}, \text{ FOR } k = 0, 1, \ldots}$$

$$\boxed{\begin{aligned} \mu_N &= a \\ \sigma_N^2 &= a \end{aligned}}$$

✓ !!!

EXAMPLE:

IN x-ray, THE NUMBER OF PHOTONS THAT ARRIVE AT A DETECTOR IN A TIME t CAN BE MODELED AS A POISSON RANDOM VARIABLE.

$$\Pr\left[X(t) = k\right] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

POISSON PROCESS

QUESTION: WHAT IS PROBABILITY THAN NO PHOTON IS DETECTED IN A TIME INTERVAL OF LENGTH t:

$$\Pr\left[X(t) = 0\right] = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = \boxed{e^{-\lambda t}}$$

EXAMPLE 3.8, P. 84 IN BOOK ⟸ IMPORTANT!

INDEPENDENT RANDOM VARIABLES:

- WE OFTEN CONSIDER MORE THAN ONE RANDOM VARIABLE AT A TIME IN IMAGING
- ~~EXAMPLE~~ A SET OF RANDOM VARIABLES ARE INDEPENDENT IF KNOWLEDGE OF SOME OF THE RANDOM VARIABLES GIVES YOU NO STATISTICAL KNOWLEDGE OF THE REMAINING VARIABLES.

SUMMING RANDOM VARIABLES:

$$\boxed{\mu_s = \mu_1 + \mu_2 + \cdots + \mu_m}$$ ⟸ DOESN'T REQUIRE INDEPENDENCE

WHAT IS VARIANCE OF NEW R.V.?

IF INDEPENDENT:

$$\boxed{\sigma_s^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_m^2}$$ ⟸ VARIANCES ADD! NOT STANDARD DEVIATIONS!

WHAT IS THE pdf OF ADDED RANDOM VARIABLES?

$$\boxed{P_s(x) = P_1(x) * P_2(x) * \dots * P_M(x)}$$

ONLY IF INDEPENDENT!

EXAMPLE:

WHAT IS THE SUM S' OF TWO INDEPENDENT GAUSSIAN RANDOM VARIABLES N_1 AND N_2? ⇐ ZERO MEAN

⇐ VARIANCE $\sigma_1^2 = \sigma_2^2 = \sigma^2$

MEAN OF S'?

VARIANCE OF S'?

pdf OF S'?

$$\mu_S = \mu_1 + \mu_2 = 0$$

$$\sigma_S^2 = \sigma_1^2 + \sigma_2^2 = 2\sigma^2$$

CONVOLUTION OF 2 GAUSSIANS YIELDS GAUSSIAN, SO:

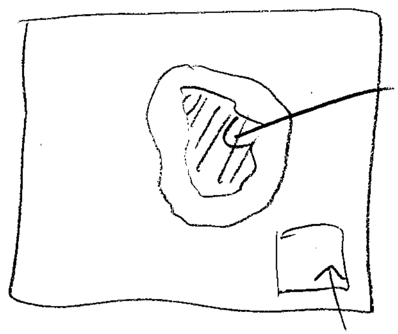
$$\boxed{P_{S'}(x) = \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\frac{x^2}{4\sigma^2}}}$$

SNR

AMPLITUDE SNR:

$$SNR_a = \frac{\text{AMPLITUDE}(\overset{\text{SIGNAL } f}{f})}{\text{AMPLITUDE(NOISE)}}$$

MRI:



$$\boxed{SNR_a = \frac{\text{AVERAGE SIGNAL}}{\sigma \text{ OF NOISE}}}$$

WHAT HAPPENS IF I AVERAGE TWO IMAGES?

EXAMPLE: PROJECTION RADIOGRAPHY:

PHOTONS G COUNTED PER UNIT AREA BY AN x-ray IMAGE INTENSIFIER (R.A.V.!) FOLLOWS A POISSON DISTRIBUTION.

LET: – SIGNAL f BE AVERAGE PHOTON COUNT PER UNIT AREA (MEAN OF G, OR μ_G!)

– NOISE N IS RANDOM VARIATION OF THIS COUNT AROUND MEAN.

$$SNR_a = \frac{\mu_G}{\sigma_G} = \frac{\mu}{\sqrt{\mu}} = \boxed{\sqrt{\mu}}\,!$$

POWER SNR!

- NOT GOING TO TALK ABOUT MUCH.
- READ UP ON NOISE POWER SPECTRUM! (p. 88)

DECIBELS:

IN DECIBELS

$$SNR_a = 20 \times \log_{10} SNR_a$$

$$SNR_p = 10 \times \log_{10} SNR_p$$

CNR (CONTRAST TO NOISE)

OR DIFFERENTIAL SNR

$$SNR_{DIFF} = CNR = \frac{SIGNAL_1 - SIGNAL_2}{\sigma_{BACKGROUND}}$$