

Midterm 1 Review Topics

Special Topics in Signals & Systems: Biomedical Imaging ECEn 682R, Section 3

The following outline follows the topics in the book and lecture notes very closely, and should give you some idea of the content I expect you to be familiar with for the midterm. As you read over it, if there are areas or terms that you aren't comfortable with, go review the relevant sections in the book, your notes, or the lecture notes.

With only a few exceptions, I don't expect you to memorize lots of complicated equations. However, I do expect you to understand what the fundamental equations we've covered in class mean, and understand how to use them. If you have any questions about the material, please bring them to the Midterm Review Session (Monday 10/19 from 5-6pm in 490 CB), stop by my office, or email me.

In addition to the material outlined here, expect some problems very similar to homework problems that you have worked. Reviewing the solution sets for Homeworks 1-5 is a good idea.

The exam will be 3 hours, closed notes, closed book, but a 3x5 card of notes (both sides) is allowed. Calculators are allowed (and will probably be needed). The exam will consist of both a multiple choice section and written problems.

Good luck!

Dr. Bangerter

1) **Basic Imaging Principles**

- a) Terminology: invasive vs. non-invasive
- b) Image orientations: sagittal, coronal, and axial (or transversal)
- c) Who discovered x-rays and when?
- d) Terminology: static vs. fluoroscopic
- e) Terminology: functional vs. structural (or anatomic) imaging

2) **Overview of Imaging Modalities**

- a) Be able to describe at a *very high level* the differences between the following non-invasive imaging modalities:
 - i) Projection Radiography
 - ii) Computer Tomography (CT)
 - iii) Nuclear Medicine
 - iv) Ultrasound
 - v) MRI

3) **Signals and Systems**

- a) Terminology: signal vs. system
- b) Signals:**
 - i) Understand the following special signals and their properties:
 - (1) Point impulse (in 1-D and 2-D) and shifted point impulses
 - (a) Sifting property ← VERY IMPORTANT
 - (b) Scaling property

- (2) Comb and sampling functions (in 1-D and 2-D)
- (3) Rect function
- (4) Sinc function
- (5) General complex exponential signals in 1-D and 2-D
- ii) Understand properties of signals:
 - (1) Separability
 - (2) Periodicity
 - (3) Linearity
- c) **Systems:**
 - i) Understand the definition of a system and the notations used in class
 - ii) Understand system properties
 - (1) Linearity: What is it, and how do we determine whether a system is linear or not?
 - (2) Shift invariance: What is it, and how do we determine whether a system is shift invariant or not?
 - (3) Stability: How do we determine if a system is BIBO stable?
 - (4) What does it mean for a system to be separable?
 - iii) Understand the convolution integral, and simple convolution properties
 - (1) Convolution of a signal with an impulse or a shifted impulse
 - iv) Linear Shift Invariant (LSI) Systems
 - (1) Understand the Point Spread Function (PSF), or impulse response
 - (2) Understand how to determine the output of an LSI system given an arbitrary input and the PSF (convolution!)
- d) **The 2D Fourier Transform**
 - i) Know the 1-D and 2-D Fourier Transform integrals and their inverses, and how to apply them to compute Fourier Transforms (when the integrals are tractable)
 - ii) What conditions guarantee existence of the FT? (If signal is both absolutely integrable and continuous)
 - iii) Know basic Fourier Transform pairs in 2-D (Table 2.1 in book) conceptually (although you don't need to memorize all of the details)
 - (1) FT of constant is delta
 - (2) FT of exponential is shifted delta
 - (3) FT of sin and cos give pairs of deltas
 - (4) FT of rect is a sinc
 - (5) FT of sinc is a rect
 - (6) FT of comb or sampling function is a comb or sampling function
 - (7) FT of a Gaussian is a Gaussian (know how to recognize a Gaussian when you see a formula for one!)
 - iv) Understand basic Fourier Transform properties at a high level
 - (1) FT is linear
 - (2) Translation \rightarrow linear phase term in Fourier domain
 - (3) Conjugation \rightarrow conjugation and time (k-space) reversal in Fourier domain
 - (4) Real-valued signals \rightarrow conjugate symmetry in Fourier domain
 - (5) Scaling \rightarrow opposite scaling in Fourier domain
 - (6) Convolution Theorem and Product Theorem
 - (7) Parseval's Theorem! (These always make good exam questions...)
- e) **LSI Systems and the Fourier Transform**
 - i) Understand what the Transfer Function (or Frequency Response) of an LSI system is, and its relationship to the Point Spread Function (PSF)
 - ii) Understand how to compute the output of an LSI system given an arbitrary input and the system's Transfer Function
- f) **Sampling**

- i) Understand the sampling signal model we derived in class (and the book) using delta-train sampling functions
- ii) Nyquist
 - (1) Understand the Nyquist Sampling Rate and the Nyquist Sampling Period
 - (2) If you know that a signal is band-limited to certain spatial frequencies, know how to determine the minimum needed sampling rate (or maximum sampling period)
 - (3) Be able to explain what an anti-aliasing filter is and why we might need one
 - (4) Understand the process for reconstructing an image from its samples

4) Image Quality and Noise

a) Contrast

- i) How do we define local contrast between two tissues?
- ii) How do we define Contrast to Noise Ratio (CNR)

b) Modulation Transfer Function (MTF)

- i) Understand the concept of the Modulation Transfer Function (MTF) of a system, and how to determine the MTF of an LSI system from its Transfer Function
- ii) Understand what the MTF is telling us about spatial resolution and contrast

c) Resolution

- i) Quantifying resolution using the FWHM of PSF
- ii) How can we relate the cutoff frequency of the MTF to resolution?
- iii) Terminology: isotropic system

d) Noise

- i) General term for random fluctuations in an image
- ii) We typically model as some kind of random variable (Gaussian, Poisson)
- iii) Review definitions of:
 - (1) Amplitude SNR
 - (2) Contrast to Noise Ratio (CNR) ← the book calls this “differential SNR”

e) Artifacts: What are they?

f) Distortion: What is it?

g) Accuracy:

- i) **Quantitative Accuracy:** We’re sometimes interested in a numerical value of a given anatomic or functional feature within an image. (i.e., tumor dimensions, glucose metabolic rate)
 - (1) Understand the difference between “systematic error” (or “bias”) and “statistical error” (or “imprecision”)
- ii) **Diagnostic Accuracy:** How good is an imaging technique at correctly identifying disease (and lack of disease).
 - (1) Sensitivity (or True-Positive Fraction): Fraction of patients with disease who test positive.
 - (2) Specificity (or True-Negative Fraction): Fraction of patients without disease who test negative.
 - (3) Diagnostic Accuracy: Fraction of patients diagnosed correctly. (Is this alone a good measure of the diagnostic performance of a technique? Why or why not?)
 - (4) Positive Predictive Value (PPV): Fraction of patients who test positive that actually have the disease.
 - (5) Negative Predictive Value (NPV): Fraction of patients who test negative that really don’t have the disease.
 - (6) Prevalence: Fraction of patients in a population with the disease.

5) Random Variables

a) Continuous Random Variables

- i) Uniquely specified by their Probability Density Function (pdf)
- ii) Terminology: mean, variance, standard deviation (understand these)

- iii) **Uniform Random Variable:**
 - (1) Understand the concept, and how to get the mean and variance
 - (2) Know what the pdf looks like
- iv) **Gaussian Random Variable:**
 - (1) Understand the bell curve concept and what the pdf looks like
 - (2) Be able to recognize the basic form of the pdf (as an equation)
 - (3) Is a Gaussian completely specified by its mean and variance?
- b) Discrete Random Variables**
 - i) Uniquely specified by a Probability Mass Function (PMF)
 - ii) **Poisson Random Variable:**
 - (1) Takes on integer values greater than or equal to 0
 - (2) Be able to recognize the basic form of the PMF
 - (3) Know that the mean and variance of a Poisson random variable are the same!
 - (4) Know that the number of x-ray photons that arrive at a detector in a given period of time can be modeled as a Poisson random variable. (We call this “quantum mottle” in x-ray and CT.)
 - (5) Understand example 3.8 in the book
 - c) Know what it means for random variables to be “independent”
 - d) Summing random variables:
 - i) What is the mean of the sum of a series of random variables?
 - ii) What is variance of the sum of a series of **independent** random variables?
 - iii) What is the pdf of the sum of a series of **independent** random variables?
- 6) **Physics of Radiography**
 - a) Radiographic imaging uses “ionizing radiation”. Understand what that is and how it is defined.
 - b) Ionizing radiation is the basis for Projection Radiography (plain vanilla x-ray) and Computed Tomography (CT)
 - c) Radiographic imaging is in general anatomical and not functional
 - d) Radiography does not involve radioactivity! (Although nuclear medicine does.)
 - e) Understand the difference between ionization and excitation
 - f) What is the energy threshold for radiation to be considered ionizing?
 - g) What is an electron volt (eV), and how is it defined?
 - h) Understand the concept of electron binding energy
 - i) **Types of Ionizing Radiation:**
 - i) Particulate Radiation**
 - (1) Review and understand formulas for relativistic and non-relativistic energy (don’t need to memorize)
 - (2) The type of particulate radiation we are most concerned with is electron beams
 - ii) Electromagnetic (EM) Radiation**
 - (1) Composed of packets or quanta of energy called photons
 - (2) Has no rest mass and no charge
 - (3) Examples include the entire electromagnetic spectrum (radio waves, microwaves, infrared light, visible light, ultraviolet light, x-rays, gamma rays)
 - (4) Review and understand formulas for energy, wavelength, frequency, etc. of EM radiation
 - (5) Know what the rough energy ranges (in keV) are for x-rays!
 - j) **Interactions of Particulate and EM Radiation with Materials**
 - i) Particulate radiation interactions come in 3 types:**
 - (1) **Collisional transfer:**
 - (a) Energetic electron glances off another electron, losing energy in the process and exciting the other electron. As excited electron returns to ground state, it emits infrared radiation, heating the object.

- (b) Sometimes the collision is direct enough that the bombarded electron is ionized, forming a new energetic electron called a “delta ray”.
 - (2) **Characteristic Radiation:**
 - (a) Energetic electron unseats a k-shell electron
 - (b) The resulting “hole” is filled by an L, M, or N-shell electron
 - (c) An x-ray photon is released (when the differences in binding energy are in the x-ray range)
 - (d) Only happens at discrete energies (corresponding to the exact difference in binding energies between electrons in different shells)
 - (3) **Bremsstrahlung Radiation:**
 - (a) Primary source of x-rays in an x-ray tube
 - (b) “Breaking radiation” that occurs when energetic electrons are slowed down as they pass by the positively-charged nucleus of an atom
 - (c) Unlike characteristic radiation, Bremsstrahlung radiation can occur at any frequency and is not limited to discrete energies
 - (4) Both Characteristic Radiation and Bremsstrahlung Radiation are termed “Radiative Transfer”
 - ii) **EM radiation interactions come in 2 important types (for our purposes)**
 - (1) **Photoelectric Effect:**
 - (a) Incident photon is completely absorbed by an atom (unlike in Compton scattering)
 - (b) Incident photon interacts with coulomb field of the nucleus of an atom, causing ejection of an electron (usually from the K-shell) called a “photoelectron”
 - (c) Also produces characteristic radiation
 - (d) Can produce a so-called “Auger” electron (another energetic electron) instead of releasing characteristic radiation
 - (2) **Compton Scattering:**
 - (a) Incident photon ejects an outer shell (or “valence”) electron, yielding energetic “Compton electron”
 - (b) Incident photon loses energy to the Compton electron, and changes directions (i.e., is scattered)
 - (c) Understand how to use the equation for the energy of a Compton-scattered photon (but don’t need to memorize)
 - (d) Understand that Compton scattering typically degrades image quality and is undesirable in medical imaging
 - k) **Attenuation of EM Radiation**
 - i) Understand the concepts of energy fluence, energy fluence rate, and intensity
 - ii) Understand the concept of the x-ray spectrum $S(E)$, and how we find the intensity of a polyenergetic source given $S(E)$
 - iii) **Linear Attenuation Coefficient**
 - (1) What is it?
 - (2) Know the fundamental photon attenuation law $\rightarrow I = I_0 * \exp(-\mu * \Delta x)$
- 7) **Projection Radiography**
- a) **Advantages:**
 - i) Short exposure time
 - ii) Large FOV
 - iii) Low cost
 - iv) Low radiation exposure
 - v) Excellent contrast and spatial resolution
 - b) **Disadvantages:**
 - i) 2-D projection instead of 2-D slice or 3-D dataset
 - ii) Ionizing radiation \rightarrow radiogenic carcinogenesis

- iii) Commonly used for: Chest x-rays (pneumonia), heart disease, lung disease, bone fractures, cancer (mammography), vascular disease, gastrointestinal imaging
- c) **X-ray Tubes:** Know how an x-ray tube works and the important parts
 - i) Cathode
 - ii) Anode
 - iii) Bremsstrahlung radiation dominates
 - iv) Cathode current controls # of emitted photons
 - v) Energy of emitted photons is controlled by the voltage between the cathode and anode (the “tube voltage”)
 - vi) Typical x-ray values for medical imaging are between 50 keV and 125 keV
 - vii) Understand and be able to recognize the general form of a typical x-ray spectrum (as shown in Figure 5.5 in the book) → characteristics include a cut-off frequency at higher energies, characteristic radiation spikes, and the general humped form of bremsstrahlung radiation.
- d) **Filtration of x-rays:**
 - i) Why do we filter?
 - ii) What is “beam hardening”?
 - iii) Why do we want to limit the amount of low-energy x-rays entering the body?
- e) **Collimation:**
 - i) What is it?
 - ii) Why do we do it?
- f) **Contrast Agents:**
 - i) What are they, and why might we want to use them?
 - ii) Typically high Z materials like Iodine ($Z = 53$) and Barium ($Z = 56$)
- g) **Controlling scatter:**
 - i) What are grids and scanning slits used for?
- h) **Detectors:**
 - i) Terminology: fluorescence vs. phosphorescence
 - ii) Intensifying screens: Understand conceptually how they work and why they are used
 - iii) Image intensifiers:
 - (1) Usually used for fluoroscopy (low dose, real time)
 - (2) Generally worse spatial resolution than film
 - (3) Increased noise and geometric distortion
 - iv) Storage phosphors:
 - (1) Used in “computed radiography”
 - (2) Use a screen coated with a storage phosphor
 - (3) Image is retrieved by pixelwise scanning with a laser
 - v) Flat panel detectors
 - (1) Typically very fast
 - (2) Used in “direct radiography”
- i) **Basic X-ray Imaging Equation**
 - i) Review Lecture 12 and the associated homework problems
- j) **Geometric Effects on Image Quality**
 - i) Understand image drop-off from inverse-square effects, obliquity, and path length
 - ii) What is the “anode heel effect”, and what causes it?
 - iii) Understand how to compute the magnification of an object in an x-ray image based on source location, location of the object, and detector location
- k) Understand penumbra and edge-blurring effects. What causes these effects?
- l) What causes film-screen blurring?
- m) **Noise in X-ray Projection Radiography**
 - i) Sources of noise:
 - (1) Poisson process arising from discrete x-ray photons: “quantum mottle”

- (2) Detector quantum efficiency and ability to localize x-ray photon arrival
 - ii) Trade-offs:
 - (1) Lower x-ray energies yield better contrast (at higher x-ray energies, the differences in μ across tissues is smaller)
 - (2) However, lower x-ray energies are absorbed more, so SNR goes down and patient radiation dose goes up
 - (3) If x-ray energies get too high, the number of x-ray photons per Roentgen decreases, and SNR is lower for a given radiation exposure. (The more photons reaching the detector, the higher the SNR.)
 - (4) You need to find a happy medium between SNR, contrast, and patient radiation dose!
 - n) **Summary of effects on x-ray resolution:**
 - i) Quality of anode tip
 - ii) Patient size \rightarrow more scattering from thicker patients
 - iii) Scattering properties of the intensifying screen
 - iv) Film resolution (mainly the grain size) \leftarrow when film is used
 - v) When image intensifier systems or storage phosphors are used, the sampling step at the end of the imaging chain usually has the largest effect on image resolution
 - (1) In computed radiography (using storage phosphors) the spot size of the laser has a large impact on resolution
 - (2) Image intensifier systems are often used in conjunction with a video camera, which has a large impact on resolution
 - o) **Biologic Effects:**
 - i) Ionizing radiation can destroy cells, cause cells to lose the ability to divide, cause cells to divide in an uncontrolled way, or cause such small damage that the cell can repair itself.
 - ii) Different organs are affected differently by ionizing radiation.
 - iii) What is the "linear no-threshold model"?
- 8) **Know both the integral and the Taylor series approximation I kept harping on in class. If you don't know what I'm referring to, ask someone. ☺**